

سوال ۱

$$\begin{aligned} \xrightarrow{x} f(-x) - xf(x) &= 2|x|+1 \xrightarrow{x \rightarrow -x} -xf(-x) + x^2 f(x) = -2|x| - x \\ \xrightarrow{-x} f(x) + xf(-x) &= 2|-x|+1 \quad xf(-x) + f(x) = 2|x|+1 \\ \rightarrow (1+x^2)f(x) &= \frac{2|-x|+1}{2|x|} - 2|x|-x \rightarrow f(x) = \frac{f(x)(-x+1)-x}{1+x^2} \end{aligned}$$

سوال ۲

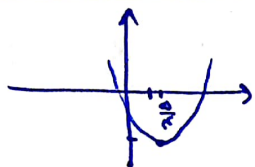
$$\begin{aligned} f(x) + f(0) &= x^2 - 3x + 4 \xrightarrow{x=0} 2f(0) = 4 \rightarrow f(0) = 2 \\ \rightarrow f(x) + 2 &= x^2 - 3x + 4 \rightarrow f(x) = x^2 - 3x + 2 \xrightarrow{x^2} f(x^2) = x^4 - 3x^2 + 2 \end{aligned}$$

سوال ۳

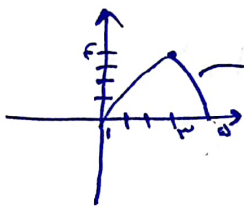
$$\begin{aligned} y &= \frac{x^2+x+2}{x-1} \rightarrow xy - y = x^2+x+2 \rightarrow x^2+(1-y)x+2+y=0 \\ x &= \frac{-1+y \pm \sqrt{(1-y)^2 - 4y-8}}{2} \xrightarrow{\Delta \geq 0} (1-y)^2 - 4y - 8 \geq 0 \rightarrow y^2 - 4y - 7 \geq 0 \quad \frac{-1}{+1} - \frac{7}{-1} + \\ R_y &= (-\infty, -1] \cup [7, +\infty) \end{aligned}$$

سوال ۴

$$\begin{aligned} \text{الف)} \quad y &= 2x^2 - 5x + 1 \quad 2\left(\frac{5a}{4}\right) - \frac{5a}{2} + 1 = \frac{5a}{2} - \frac{5a}{2} + \frac{1}{2} = \frac{-1}{2} \\ x_2 &= \frac{5}{4} \quad R_y = \left[\frac{-1}{2}, +\infty\right) \end{aligned}$$

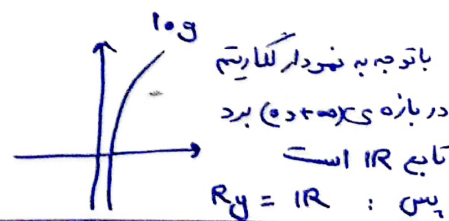


$$\begin{aligned} \text{ب)} \quad y &= \sqrt{-x^2 + 4x - 5} \quad \begin{matrix} \text{برای } x \leq 5 \\ \text{و } x \geq 1 \end{matrix} \\ \frac{-4}{-2} &= 2 \quad -9 + 18 - 5 = 4 \quad R = [0, 4] \xrightarrow{\sqrt{}} R_y = [0, 2] \end{aligned}$$



$$\text{ج)} \quad y = \sqrt{x^3 - 3x^2 + 6x + 1} \quad \begin{matrix} \text{تابع چند جمله} \\ R = \mathbb{R} \end{matrix} \xrightarrow{\sqrt{}} R_y = [0, +\infty)$$

$$\begin{aligned} \text{د)} \quad y &= \log(x^3 - 4x^2 + 7x + 4) \\ \log &\text{ بر روی } \mathbb{R} \text{ تعریف می شود} \rightarrow \text{از این بازه گاریم} \rightarrow (0, +\infty) \end{aligned}$$



$$\text{الف)} \quad y = \frac{3x+1}{2x-4} \quad R = \mathbb{R} - \text{مجاانب انتی} \rightarrow R_y = \mathbb{R} - \left\{\frac{3}{2}\right\}$$

سوال ۵

$$\text{ب)} \quad y = \sqrt{\frac{4x+1}{x-2}} \rightarrow R = \mathbb{R} - \text{مجاانب انتی} \rightarrow R = \mathbb{R} - \{2\} \xrightarrow{\sqrt{}} R_y = [0, 2) \cup (2, +\infty)$$

$$\text{ج)} \quad y = \frac{x^2+6}{\sqrt{x^2+3}} = \frac{x^2+3+3}{\sqrt{x^2+3}} = \underbrace{\sqrt{x^2+3}}_a + \underbrace{\frac{3}{\sqrt{x^2+3}}}_{\frac{1}{a}} \xrightarrow{\min(\sqrt{x^2+3}) = \sqrt{3}} \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$$\begin{aligned} R_y &= \left[\frac{4}{\sqrt{3}}, +\infty\right) \\ \text{د)} \quad y &= x^2 + \frac{1}{x^2+4} \xrightarrow{\pm 4} x^2+4 + \frac{1}{x^2+4} - 4 \xrightarrow{\min(x^2+4)=4} 4 + \frac{1}{4} - 4 \rightarrow R_y = \left[\frac{1}{4}, +\infty\right) \end{aligned}$$

سوال ۶

$$y = \frac{3x^2 + 4}{x^2 - 1} \xrightarrow{x \rightarrow \infty} -4 \quad \xrightarrow{x \rightarrow -\infty} 3$$

چون خروج صفری شود
بر د خارج از این عدد است

$$R_y = (-\infty, -4] \cup (3, +\infty)$$

$$y = \frac{3x^2 + 4}{x^2 - 1} \rightarrow yx^2 - y = 3x^2 + 4 \rightarrow (y - 3)x^2 = 4 + y \rightarrow x^2 = \frac{4 + y}{y - 3}$$

$$x = \pm \sqrt{\frac{4 + y}{y - 3}} \quad \xrightarrow{\frac{-4}{-1} = 4} \quad R_y = (-\infty, -4] \cup (3, +\infty)$$

سوال ۷

$$f(x) = \frac{2 \sin x + 4}{\sin x - 3}$$

چون خروج صفری شود
پس بر دین این عدد است

$$R_f = [-3, -\frac{1}{2}]$$

سوال ۸

$$y = \left| \frac{3x - 1}{4x} \right|$$

$$R_y = (0, +\infty) - \left\{ \frac{3}{4} \right\}$$

$$y = \sqrt{\frac{9x - 18}{-8 + 4x}} \rightarrow \sqrt{\frac{9(x-2)}{4(x-2)}} = \frac{3}{2} \quad R_y = \left\{ \frac{3}{2} \right\}$$

سوال ۹

$$y = x|x| - 2x$$

$$R_y = \mathbb{R}$$

سوال ۱۰

$$f(1-x) + 1$$

$$y = |f(x)| + 1$$