

$$f(-n) = n f(n) + r |n| + 1 \quad \begin{cases} n=n \rightarrow f(-n) - n f(n) = r |n| + 1 \quad \text{یا} \quad (-n) \rightarrow n f(-n) + n^2 f(n) = (r |n| + 1) \cdot n \\ n=-n \rightarrow f(n) + n f(-n) = r |n| + 1 \end{cases}$$

$$\rightarrow (n^2 + 1) f(n) = (r |n| + 1) + (r |n| + 1) \cdot n \rightarrow f(n) = \frac{(n+1)(r |n| + 1)}{n^2 + 1}$$

$$f(n) + f(0) = n^2 - 3n + 4 \quad \xrightarrow{n=0} f(0) = 4 \rightarrow f(0) = 4$$

$$f(n) = n^2 - 3n + 4 \rightarrow f(n^2) = n^4 - 3n^2 + 4$$

$$y = \frac{n^2 + n + r}{n - 1} \rightarrow y n - y = n^2 + n + r \rightarrow n^2 + (1 - y) n + (r + y) = 0 \quad \Delta = b^2 - 4ac$$

$$\Delta = (1 - y)^2 - 4(1)(r + y) = 1 + y^2 - 2y - 4r - 4y = y^2 - 4y - 4r - 3$$

$$\Delta \geq 0 \rightarrow y^2 - 4y - 4r - 3 \geq 0 \rightarrow \begin{cases} y = -1 \\ y = 5 \end{cases}$$

$$\frac{y}{+} \quad \frac{-1}{+} \quad \frac{5}{-} \quad \frac{r}{+}$$

$$R_f: (-\infty, -1] \cup [5, +\infty)$$

الف) $y = 2n^2 - 2n + 1$ ب) $y = \sqrt{-n^2 + 4n - 5}$ ج) $y = \sqrt{n^2 - 4n^2 + 5n + 1}$

$x_{\min} = \frac{-b}{2a} = \frac{1}{2} \quad R_f: [\frac{1}{2}, +\infty)$ $\begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{\max} = -x + \sqrt{x^2 - 4} = 2 \end{cases} \quad R_f: [1, 2]$

$y_{\min} = f(\frac{1}{2}) = \frac{1}{4} - \frac{1}{2} + \frac{1}{2} = \frac{1}{4}$ $(-\infty, 2] \rightarrow R_f: [0, 2]$

د) $y = \lg(n^2 - 4n^2 + 5n + 1) \rightarrow y = \lg(\mathbb{R}) \rightarrow R_f: \mathbb{R}$

الف) $y = \frac{n^2 + 1}{n - 4} \quad R_f: \mathbb{R} - \{4\}$ ب) $y = \frac{n^2 + 1}{n - 2} \quad R_f: [2, +\infty) - \{2\}$

ج) $y = \frac{n^2 + 1}{\sqrt{n^2 + 1}} = \sqrt{n^2 + 1} + \frac{1}{\sqrt{n^2 + 1}} \rightarrow \sqrt{1 + \frac{1}{n^2}} + \frac{1}{\sqrt{1 + \frac{1}{n^2}}} = \frac{2}{\sqrt{1 + \frac{1}{n^2}}} \rightarrow R_f: [\frac{2}{\sqrt{2}}, +\infty)$

د) $y = \frac{n^2 + 1}{n^2 + 1} - 1 = 0 \rightarrow R_f: [\frac{1}{2}, +\infty)$

$y = \frac{r \sin^2 x + \varepsilon}{x^2 - 1} \neq 0$ روش اولی: $x^2 = -\frac{y - \varepsilon}{y - r} \rightarrow x = \pm \sqrt{\frac{y + \varepsilon}{y - r}}$ $y \mid \frac{-\varepsilon}{+1} - \frac{r}{-1} +$
 $R_f: (-\infty, -\varepsilon) \cup (r, +\infty)$
 (۲)

(۲) در (۲):
 $0 = x^2 \rightarrow x = 0$
 $x = \pi^2 \rightarrow y = \frac{r}{-1} = -r$
 $x = \pi^2 \rightarrow y = \frac{r(+\infty)}{(+\infty)} = r$
 $R_f: (-\infty, -\varepsilon) \cup (r, +\infty)$
 (۲)

$x^2 - 1 \neq 0 \rightarrow x \neq \pm 1$
 خارج ناحیه
 ممنوعه
 (۲)

$f(x) = \frac{r \sin x + \varepsilon}{\sin x - r} \neq 0 \rightarrow \sin x \neq r$
 برد تابع همیشه داخل $\rightarrow$ خارج نمی‌تواند صورت شود
 $(\sin x)_{\min} = -1 \rightarrow f(x) = \frac{-r + \varepsilon}{-r} = -\frac{1}{r}$
 $(\sin x)_{\max} = 1 \rightarrow f(x) = \frac{r + \varepsilon}{1 - r} = \frac{r}{1 - r} < -r \Rightarrow R_f: [-r, -\frac{1}{r}]$
 (۲)

$y = \left| \frac{r \sin x - 1}{\varepsilon \sin x} \right|$ $\rightarrow \frac{r \sin x - 1}{\varepsilon \sin x}$
 (۲) $\rightarrow \sqrt{\frac{a \sin x - 1}{-1 + \varepsilon \sin x}} = \sqrt{\frac{a(\sin x - r)}{r(\sin x - r)}} \rightarrow x \neq r$
 $y = \sqrt{\frac{a}{\varepsilon}} = \frac{r}{r}$
 $R_f: \left[\frac{r}{\varepsilon}, +\infty \right)$
 (۲)

$y = x|x| - r$
 $y = -x^2 - r$ $y = x^2 - r$
 $\begin{cases} x_{\min} = -\frac{b}{2a} = -\frac{r}{-2} = -\frac{r}{2} \\ y_{\max} = -(-1) - r(-1) = 1 \end{cases}$ $\begin{cases} x_{\min} = -\frac{b}{2a} = -\frac{r}{2} \\ y_{\min} = 1 - r = -1 \end{cases}$
 $R_f: \left[\frac{r}{\varepsilon}, +\infty \right)$
 (۲)

$y = f(|x|) =$ $\rightarrow f(|x|) + 1 =$
 $y = |f(|x|) + 1|$
 (۲)