

دانشگاه تهران

19

تاریخ سوال

کلاس و مدرسه

$$f(n) = ?$$

$$f(-n) - 2f(n) = 2|n| + 1 \quad (1)$$

$$n \rightarrow f(-n) - 2f(n) = 2n + 1$$

$$-n \rightarrow f(n) + 2f(-n) = 2|n| + 1$$

$$\begin{cases} 2f(-n) - 2^2 f(n) = 2n^2 + 2n \\ f(n) + 2f(-n) = 2n + 1 \end{cases} \quad (1)$$

$$(1+2^2)f(n) = 2n^2 - 2n + 1$$

$$f(n) = \frac{2n^2 - 2n + 1}{1+2^2}$$

صفحه آخر بابت نوشتن

در روش بزرگتر
در قدر مطلق در روش کوچکتر

باید سهوا

$$f(n) + f(0) = 2^2 - 2n + 4$$

$$f(2^2) = ?$$

$$n \rightarrow f(0) = 4 \rightarrow f(1) = 3$$

$$\rightarrow f(n) + 3 = 2^2 - 2n + 4 \rightarrow f(n) = 2^2 - 2n + 1$$

$$f(2^2) = 2^4 - 2 \cdot 2^2 + 1$$

$$n+1 \quad y = \frac{2^2 + 2 + 1}{2-1}$$

$$y_{n+1} - y = 2^2 + 2 + 1$$

$$y_2 - y_1 = 2^2 + 1 + y$$

$$2(y-1) = 2^2 + 1 + y \rightarrow 2^2 - (y-1) + 1 + y \rightarrow$$

$$2 = \frac{y+1 \pm \sqrt{(y-1)^2 - 4(2+y)}}{2}$$

$$\rightarrow (y-1)^2 - 4(2+y) \geq 0$$

$$y^2 - 2y + 1 - 8 - 4y \geq 0$$

$$y^2 - 6y - 7 \geq 0$$

$$(y-7)(y+1) \geq 0$$

$$\begin{array}{c|c|c} -1 & & 7 \\ \hline + & - & + \end{array}$$

$$R_f = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = 2x^2 - 2x + 1 \quad \min \left| \frac{b}{2a} = \frac{a}{4} \right|$$

$$2 \times \frac{20}{14} - \frac{20}{2} + 1 = \frac{20 - 20 + 1}{1} = \frac{-1}{1}$$

$$R_f = \left[\frac{-1}{1}, +\infty \right)$$

$$\text{ب) } y = \sqrt{-x^2 + 4x - 8}$$

$$\max \left| \frac{b}{2a} = \frac{-4}{-2} = 2 \right|$$

$$-9 + 16 - 8 = -1$$

$$R_f = \sqrt{(-\infty, 2]} = [0, 2]$$

$$\text{ج) } y = \sqrt{x^2 - 2x^2 + (2x+1)}$$

$$R_f = \sqrt{(-\infty, +\infty)} = [0, +\infty)$$

Arman

$$7) y = \log(x^2 - 4x^2 + 12x + 5) \quad R_f = R = (-\infty, +\infty)$$

$$a) y = \frac{x+1}{x-1} \quad R_f = R - \left\{ \frac{1}{2} \right\} = R - \left\{ \frac{1}{2} \right\} \quad (2) \quad \textcircled{a}$$

$$b) y = \sqrt{\frac{x+1}{x-1}} \quad R_f = \sqrt{R - \left\{ \frac{1}{2} \right\}} = [0, +\infty) - \left\{ \frac{1}{2} \right\}$$

$$c) y = \frac{x^2 + 5}{\sqrt{x^2 + 4}} = \frac{x^2 + 4 + 1}{\sqrt{x^2 + 4}} = \sqrt{x^2 + 4} + \frac{1}{\sqrt{x^2 + 4}} \quad \frac{1}{\sqrt{x^2 + 4}} \rightarrow \frac{1}{\sqrt{4}} = \frac{1}{2}$$

$$\Rightarrow R_f = \left[\frac{1}{2}, +\infty \right)$$

$$d) y = x^2 + \frac{1}{x^2 + 5} + 5 - 5 = x^2 + 5 + \frac{1}{x^2 + 5} - 5 \quad \frac{1}{x^2 + 5} \rightarrow \frac{1}{5} - 5 = -\frac{24}{5}$$

$$R_f = \left[\frac{1}{5}, +\infty \right)$$

نوع دالة $y = \frac{ax+b}{cx+d}$ دالة كسرية من الدرجة الأولى. $y = \frac{x^2 + 5}{x^2 - 1}$ دالة كسرية من الدرجة الثانية.

$$y = \frac{x^2 + 5}{x^2 - 1} \quad \text{نوع دالة كسرية من الدرجة الثانية}$$

$$y(x^2 - 1) = x^2 + 5 \quad yx^2 - y = x^2 + 5 \quad yx^2 - x^2 - y - 5 = 0 \quad (x^2 - 1)(y - 1) = 0 \quad x^2 - 1 = 0 \quad x = \pm 1$$

$$y - 1 = 0 \quad y = 1$$

$$R_f = (-\infty, -1] \cup (1, +\infty) \quad + \quad - \quad - \quad +$$

$$\left[\begin{array}{l} x=0 \rightarrow y = \frac{5}{-1} = -5 \\ x \rightarrow \infty \rightarrow y = \frac{1}{1} = 1 \end{array} \right]$$

$$\Rightarrow (-\infty, -5] \cup (1, +\infty)$$

$$f(x) = \frac{x \sin x + 5}{\sin x - 1} \quad \left[\begin{array}{l} \sin x = -1 \rightarrow y = \frac{-1 + 5}{-1 - 1} = \frac{4}{-2} = -2 \\ \sin x = 1 \rightarrow y = \frac{1 + 5}{1 - 1} = \frac{6}{0} = \infty \end{array} \right]$$

$$R_f = \left[-2, \frac{1}{2} \right)$$

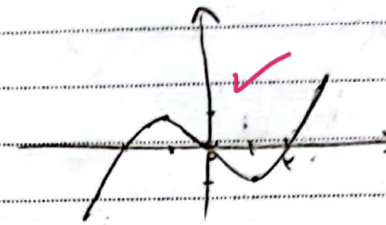
$$a) y = \left| \frac{x-1}{x} \right| \quad y = \frac{x-1}{x} \quad y = 1 - \frac{1}{x} \quad R_f = [0, +\infty)$$

$$b) y = \sqrt{\frac{9x-1}{-1+x}} = \sqrt{\frac{9(2A)}{1-(2A)}} = \sqrt{\frac{9}{1-2A}} = \frac{3}{\sqrt{1-2A}} \quad R_f = \left\{ \frac{3}{\sqrt{1-2A}} \right\}$$

Arman

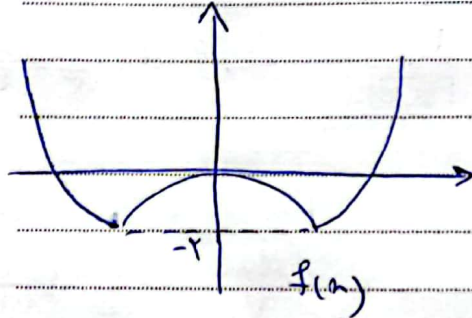
$$y = 2|x| - 1$$

$$\begin{aligned} x > 0 &\rightarrow 2x - 1 \\ x = 0 &\rightarrow 0 \\ x < 0 &\rightarrow -2x - 1 \end{aligned}$$



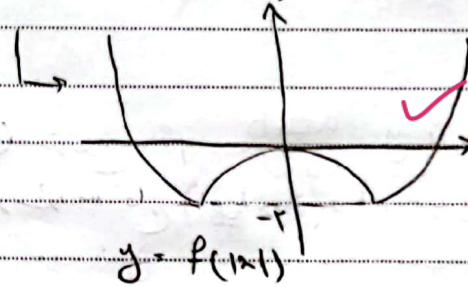
9
(2)

$$R_f = 1R$$

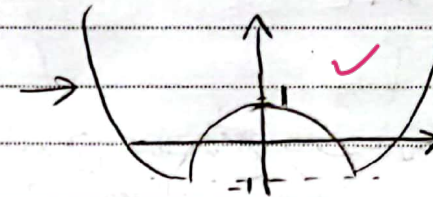


$$y = |f(x)| + 1$$

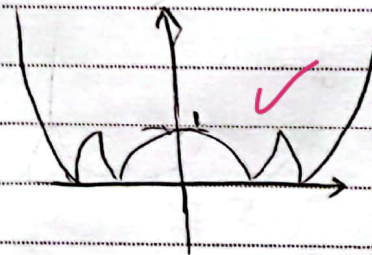
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(2)



$$y = f(x)$$



$$y = f(x) + 1$$



$$y = |f(x)| + 1$$

۱- جای n ، $-n$ قرار می دهیم و یک دستگاه می سازیم:

$$\begin{aligned} f(-n) - n f(n) &= 2|n| + 1 \xrightarrow{\times n} \begin{cases} n f(-n) - n^2 f(n) = 2n|n| + n & (I) \\ f(n) + n f(-n) = 2|n| + 1 & (II) \end{cases} \\ f(n) + n f(-n) &= 2|n| + 1 \end{aligned}$$

$$\underline{II - I} \rightarrow (1 + n^2) f(n) = 2(1 - n)|n| + 1 - n \Rightarrow f(n) = \frac{(1 - n)(2|n| + 1)}{1 + n^2}$$