

A function

f(x) = x^2 + 1

Domain: R

$$f(x) = ?$$

$$f(-x) - 2f(x) = x^2 + 1 \quad (1)$$

$$x \rightarrow f(-x) - 2f(x) = x^2 + 1$$

$$-x \rightarrow f(x) + 2f(-x) = x^2 + 1$$

$$\begin{cases} x^2 & \left\{ \begin{aligned} 2f(-x) - 2^2(f(x)) &= x^2 + 1 \\ f(x) + 2f(-x) &= x^2 + 1 \end{aligned} \right.$$

$$(1+2^2)f(x) = x^2 + 1$$

$$f(x) = \frac{x^2 + 1}{1+2^2}$$

$$f(x) + f(0) = x^2 - 2x + 4$$

$$f(2) = ?$$

(P)

$$x=0 \rightarrow f(0) = 4 \rightarrow f(0) = 4$$

$$\rightarrow f(x) + 4 = x^2 - 2x + 4 \rightarrow f(x) = x^2 - 2x$$

$$f(2) = 2^2 - 2 \cdot 2 = 0$$

$$n^{\text{th}} \quad y = \frac{x^2 + x + 1}{x-1}$$

$$y_1 - y = x^2 + x + 1$$

$$y_2 - x = x^2 + x + 1$$

$$x(y-1) = x^2 + x + 1 \rightarrow x^2 - (y-1)x + x + 1 = 0$$

$$x = \frac{y+1 \pm \sqrt{(y-1)^2 - 4(y+1)}}{2}$$

$$\rightarrow (y-1)^2 - 4(y+1) \geq 0$$

$$y^2 - 2y + 1 - 4y - 4 \geq 0$$

$$y^2 - 6y - 3 \geq 0$$

$$(y-4)(y+1) \geq 0$$

$$\begin{array}{c} -1 \\ + \end{array} \quad \begin{array}{c} -4 \\ + \end{array}$$

$$R_f = (-\infty, -1] \cup [4, +\infty)$$

$$\text{a) } y = x^2 - 2x + 1 \quad \min \left| \frac{b}{a} = \frac{1}{1} \right.$$

$$\left| \frac{1 \times 1}{1} - \frac{1 \times 0}{2} + 1 = \frac{1 - 0 + 1}{1} = \frac{2}{1} = 2 \right.$$

$$R_f = \left[\frac{-1}{1}, +\infty \right)$$

$$\text{b) } y = \sqrt{-x^2 + 4x - 8}$$

$$\max \left| \frac{b}{a} = \frac{-4}{-1} = 4 \right.$$

$$-9 + 16 - 8 = -1$$

$$R_f = \sqrt{(-\infty, 4]} = [0, 4]$$

$$\text{c) } y = \sqrt{x^2 - 2x + 1}$$

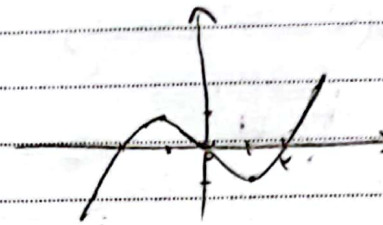
$$R_f = \sqrt{(-\infty, +\infty)} = [0, +\infty)$$

Arman

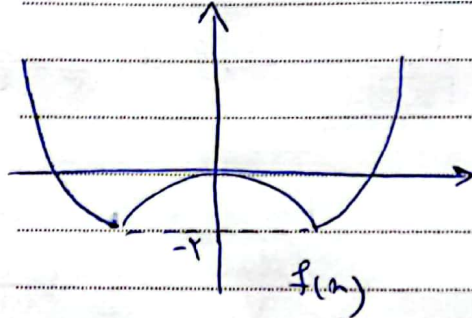
$$y = 2|x| - 1$$

$x > 0$	$2x - 1$
$x = 0$	-1
$x < 0$	$-2x - 1$

$$R_f = \mathbb{R}$$

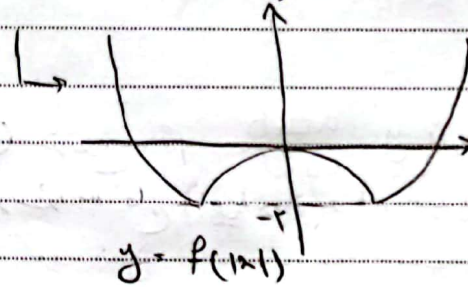


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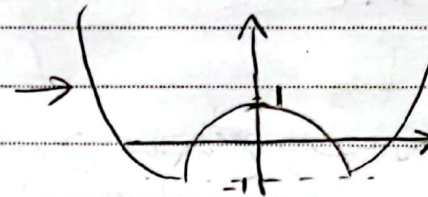


$$y = |f(x)| + 1$$

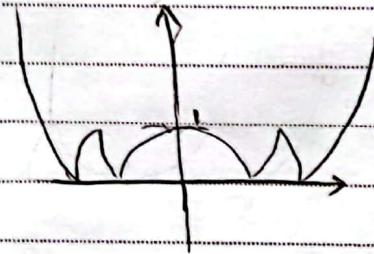
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$$y = f(x)$$



$$y = f(x) + 1$$



$$y = |f(x)| + 1$$