

$$f(x) - xf(x) = 2|x| + 1 \xrightarrow{(I)} f(x) - x(2|x| + 1) + x^2 f(x) = 2|x| + 1 \rightarrow f(x)(1+x^2) = (2|x| + 1) + x(2|x| + 1) \Rightarrow f(x) = \frac{(1+x)(2|x| + 1)}{1+x^2}$$

$$f(-x) + xf(x) = 2|x| + 1 \rightarrow f(-x) = 2|x| + 1 - xf(x) \quad (I)$$

$$f(x) + f(0) = x^2 - 2x + 4 \xrightarrow{x=0} 2f(0) = 4 \rightarrow f(0) = 2$$

$$f(x) = x^2 - 2x + 4 \rightarrow f(x^2) = x^4 - 2x^2 + 4$$

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2 \rightarrow x^2 + (1-y)x + 2+y = 0 \rightarrow x = \frac{y-1 \pm \sqrt{(1-y)^2 - 4(2+y)}}{2} \Rightarrow 1+y^2-2y-8-4y \geq 0$$

$$y^2 - 4y - 7 \geq 0 \quad \begin{matrix} - \\ + \end{matrix} \quad \begin{matrix} - \\ + \end{matrix} \quad \begin{matrix} - \\ + \end{matrix} \Rightarrow R = \mathbb{R} - (-1, 7)$$

$$\text{الف) } -\frac{\Delta}{K_a} = -\frac{2a-1}{1} = -\frac{11}{1} \rightarrow R = [-\frac{11}{1}, +\infty) \quad \text{ب) } -\frac{34-20}{-4} = 4 \rightarrow R = [0, 2]$$

$$\text{ج) } \text{توان فرد: } R = \mathbb{R} \xrightarrow{\text{انکسار}} R = [0, +\infty) \quad \text{د) } \text{توان فرد: } R = \mathbb{R} \xrightarrow{\text{انکسار}} (0, +\infty) \Rightarrow R = \mathbb{R}$$

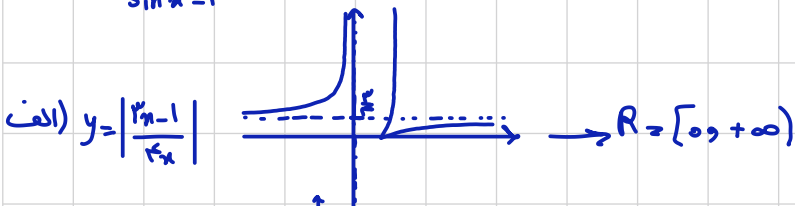
$$\text{الف) } y = \frac{3x+1}{2x-4} \rightarrow R = \mathbb{R} - \left\{\frac{3}{2}\right\} \quad \text{ب) } \mathbb{R} - \{4\} \xrightarrow{\text{انکسار}} R = [0, +\infty) - \{4\}$$

$$\text{ج) } \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \xrightarrow{a+\frac{1}{a}} \min a = \sqrt{3} \Rightarrow R = \left[\frac{\sqrt{3}}{\sqrt{3}}, +\infty\right) \quad \text{د) } x^2+4 + \frac{1}{x^2+4} - 4 \rightarrow R = \left[\frac{1}{4}, +\infty\right)$$

$$y = \frac{3x^2+4}{x^2-1} \rightarrow yx^2 - y = 3x^2 + 4 \rightarrow yx^2 - 3x^2 = y + 4 \rightarrow x^2 = \frac{y+4}{y-3} \rightarrow x = \pm \sqrt{\frac{y+4}{y-3}} \quad \begin{matrix} - \\ + \end{matrix} \quad \begin{matrix} - \\ + \end{matrix} \quad \begin{matrix} - \\ + \end{matrix} \rightarrow R = \mathbb{R} - (-3, 4]$$

$$x = +\infty \rightarrow y = 3, x = 0 \rightarrow y = -4 \xrightarrow{\text{نقطه میانی}} R = (-\infty, -4] \cup (3, +\infty)$$

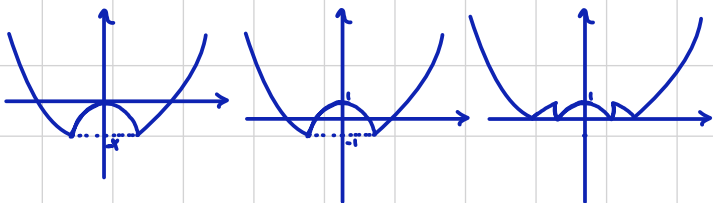
$$f(x) = \frac{2 \sin x + 4}{\sin x - 3} \rightarrow \sin x = 1 : f(x) = -3, \sin x = -1 : f(x) = -\frac{1}{4} \xrightarrow{\text{نقطه میانی}} R = \left[-3, -\frac{1}{4}\right]$$



$$\text{الف) } y = \left| \frac{x-1}{x} \right| \rightarrow R = [0, +\infty)$$

$$\text{ب) } y = \sqrt{\frac{9x-11}{-1+5x}} \rightarrow \frac{9(x-2)}{5(x-2)} = \frac{9}{5} \rightarrow y = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}} \quad R = \left\{\frac{3}{\sqrt{5}}\right\}$$

$$y = \begin{cases} x^2 - 2x & x \geq 0 \\ -x^2 - 2x & x \leq 0 \end{cases} \Rightarrow R = \mathbb{R}$$



$$f(x) = f(|x|) \quad f(|x|) + 1 \quad |f(|x|) + 1|$$