

$$\begin{cases} f(-x) - xf(x) = 2|x|+1 & \times (-x) \\ f(x) + xf(-x) = 2|-x|+1 & \times 1 \end{cases} \rightarrow \begin{cases} -xf(-x) + x^2 f(x) = -x(2|x|+1) \\ f(x) + xf(-x) = 2|x|+1 \end{cases}$$

$$f(x)(1+x^2) = (1-x)(2|x|+1)$$

$$f(x) = \frac{(1-x)(2|x|+1)}{1+x^2}$$

$$x=0: f(0) + f(0) = 0^2 - 2(0) + 1 \rightarrow 2f(0) = 1 \rightarrow f(0) = \frac{1}{2}$$

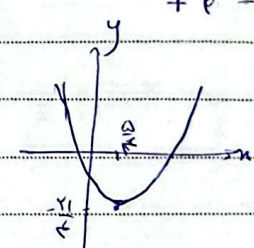
$$f(x^2) + f(0) = (x^2)^2 - 2(x^2) + 1 \rightarrow f(x^2) = x^4 - 2x^2 + 1 - \frac{1}{2} = x^4 - 2x^2 + \frac{1}{2}$$

$$y(x-1) = x^2 + x + 1 \rightarrow x^2 + (1-y)x + (1+y) = 0, \Delta = (1-y)^2 - 4(1+y) \geq 0$$

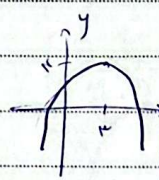
$$\Delta \geq 0 \rightarrow \Delta = 1 + y^2 - 2y - 4 - 4y = y^2 - 6y - 3 \geq 0 \rightarrow (y-3)(y+1) \geq 0$$

$$\rightarrow R = (-\infty, -1] \cup [3, +\infty)$$

الف) $\text{ent} \left| \begin{aligned} -\frac{b}{2a} &= \frac{0}{2} = 0 \\ y &= 2\left(\frac{0}{2}\right) - \frac{0}{2} + 1 = -\frac{0}{2} + 1 = \frac{1}{2} \end{aligned} \right.$

$$R = \left[-\frac{1}{2}, +\infty\right)$$


ب) $\text{ent} \left| \begin{aligned} -\frac{b}{2a} &= -\frac{4}{2} = -2 \\ y &= -9 + 11 - 4 = -2 \end{aligned} \right.$

$$R = [-2, +\infty)$$


ج) $y = \sqrt{x^2 - 4x^2 + 4x + 1} \rightarrow R_y = [0, +\infty)$

د) $y = \log(x^2 - 4x^2 + 4x + 1) \rightarrow R_y = \mathbb{R}$

هـ) $R = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \left\{\frac{1}{2}\right\}$

و) $\frac{f(n+1)}{n-2} \rightarrow R = \mathbb{R} - \{2\} \rightarrow R_y = [0, +\infty) - \{2\}$

ز) $y = \frac{x^2 + 1}{\sqrt{x^2 + 1}} = \sqrt{x^2 + 1} + \frac{1}{\sqrt{x^2 + 1}} \rightarrow R = \left[\sqrt{1} + \frac{1}{\sqrt{1}}, +\infty\right) = \left[\frac{\sqrt{4}+1}{\sqrt{1}}, +\infty\right)$

لا بد من $a + \frac{1}{a} \geq 2$

Arman

د) $y = \frac{n^r + c + 1}{n^r + c} = \frac{1}{1 - \frac{c}{n^r + c}}$ $R = \left[\frac{1}{1 - \frac{c}{n^r + c}}, +\infty \right) = \left[\frac{1}{1 - \frac{c}{n^r + c}}, +\infty \right)$
 الحد $\lim_{n \rightarrow \infty} \frac{1}{1 - \frac{c}{n^r + c}} = 1 + \frac{c}{n^r + c}$

نقطة: $\frac{n^r + \infty}{n^r + c} \rightarrow y = 1$ $n^r = 1 \rightarrow n^r = 1 \rightarrow n = \pm 1$
 $\frac{n^r}{n^r + c} \rightarrow y = -\frac{c}{n^r + c}$ $R = (-\infty, -\frac{c}{n^r + c}] \cup (1, +\infty)$

نقطة: $yn^r - y = rn^r + c \rightarrow (y-r)n^r = y + c \rightarrow n^r = \frac{y+c}{y-r} \rightarrow \frac{n^r}{y-r} = \frac{y+c}{y-r}$

$\frac{-c}{1 - \frac{c}{n^r + c}} \rightarrow R = (-\infty, -\frac{c}{n^r + c}] \cup (1, +\infty)$

$\sin n^r = 1 \rightarrow f(n) = \frac{y+c}{1-r} = \frac{y}{1-r} = -\frac{1}{r}$ $\sin n^r = -1 \rightarrow \sin n^r = -1$

$\sin n^r = 1 \rightarrow f(n) = \frac{y+c}{1-r} = \frac{y}{1-r} = -\frac{1}{r}$ $R = \left[-\frac{1}{r}, -r \right]$

ج) $y = \frac{n^r - 1}{n^r} \rightarrow R = \left\{ \frac{r}{r} \right\} \rightarrow y = \left| \frac{n^r - 1}{n^r} \right| \rightarrow R = [0, +\infty) - \left\{ \frac{r}{r} \right\}$

د) $y = \sqrt{\frac{q(n-r)}{r(n-r)}} \rightarrow y = \sqrt{\frac{q}{r}} = \frac{r}{r} \rightarrow R = \left\{ \frac{r}{r} \right\}$

$n > 0 \rightarrow \ln n \rightarrow y = n^r \ln n - \ln n \rightarrow \frac{1}{n^r} = 1$
 $n < 0 \rightarrow \ln n \rightarrow y = -n^r - \ln n \rightarrow \frac{1}{n^r} = -1$
 $R = \mathbb{R}$

