

$$1) f(-x) - xf(x) = 2|x| + 1 \rightarrow f(x) = ?$$

$$f(x) + xf(-x) = 2|x| + 1$$

$$f(-x) - xf(x) = 2|x| + 1 \rightarrow f(-x) = 2|x| + 1 + xf(x)$$

$$f(x) + xf(-x) = 2|x| + 1 \rightarrow f(x) + x(2|x| + 1 + xf(x)) = 2|x| + 1$$

$$(1+x^2)f(x) + 2x|x| + x = 2|x| + 1$$

$$(1+x^2)f(x) = 2|x| + 1 - x$$

$$(1+x^2)f(x) + x(2|x| + 1) - (2|x| + 1) = (1+x^2)f(x) + (x-1)(2|x| + 1) \rightarrow (1+x^2)f(x) = -(x-1)(2|x| + 1)$$

$$\rightarrow (1+x^2)f(x) = (1|x| + 1)(1-x) \rightarrow f(x) = \frac{(2|x| + 1)(1-x)}{1+x^2}$$

$$2) f(x) + f(x^2) = x^2 - 3x + 4 \rightarrow f(x^2) = ?$$

$$x=0 \rightarrow f(0) = 0 - 0 + 4 = 4 \rightarrow f(0) = 4$$

$$f(x) + 4 = x^2 - 3x + 4 \rightarrow f(x) = x^2 - 3x \rightarrow f(x^2) = (x^2)^2 - 3(x^2) + 4$$

$$\rightarrow f(x^2) = x^4 - 3x^2 + 4$$

$$3) R_f = ? \quad y = \frac{x^2 + x + 2}{x-1} \quad \text{مقسوم عليه}$$

$$y(x-1) = x^2 + x + 2 \rightarrow xy - y = x^2 + x + 2$$

$$x^2 + x - xy + y + 2 = 0 \rightarrow x^2 + (1-y)x + (y+2) = 0 \xrightarrow{\Delta \geq 0} (1-y)^2 - 4(1)(y+2) \geq 0$$

$$1 - 2y + y^2 - 4y - 8 \geq 0 \rightarrow y^2 - 6y - 7 \geq 0 \rightarrow (y-7)(y+1) \geq 0$$

$$R = (-\infty, -1] \cup [7, +\infty)$$

$$4) \text{ ا) } y = \frac{x^2 - 2x + 1}{x^2 + 4x - 4} \rightarrow \frac{\Delta}{f} \rightarrow y = \frac{2}{x^2 + 4x - 4} - 2\left(\frac{2}{x^2 + 4x - 4}\right) + 1 = -\frac{1}{x^2 + 4x - 4}$$

$$\rightarrow y = \frac{-1}{x^2 + 4x - 4} \rightarrow -(-x^2 - 4x + 4) \xrightarrow{\Delta \geq 0} (x-3)^2 - 4 \xrightarrow{\Delta \geq 0} -(x-3)^2 \geq 0$$

$$0 \leq -(x-3)^2 \leq 0 \rightarrow [0, 0]$$

$$2) y = \sqrt{x^2 - 4x + 4} + 1 \rightarrow \mathbb{R} \rightarrow \sqrt{\mathbb{R}} = [0, +\infty)$$

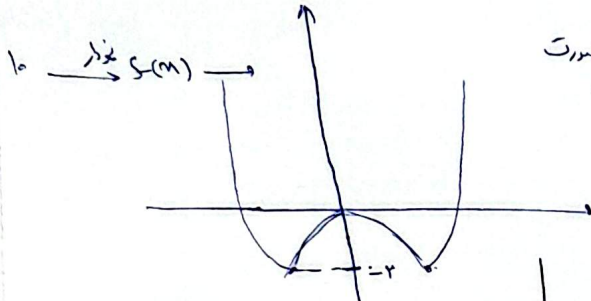
$$3) y = \log(x^2 - 4x + 4) \rightarrow \mathbb{R}$$

9) $y = x/|x| - 2x \rightarrow R_f = ?$ $\min = -1$

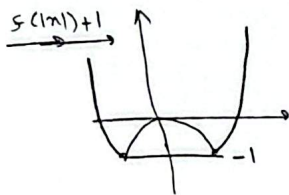
$\rightarrow x \geq 0 \rightarrow |x| = x \rightarrow y = x^2 - 2x \xrightarrow{\text{در این قسمت}} (x-1)^2 - 1 = y \rightarrow [-1, +\infty)$

$\rightarrow x \leq 0 \rightarrow |x| = -x \rightarrow y = -x^2 - 2x \Rightarrow y = -(x+1)^2 + 1 \xrightarrow{x = -1 \rightarrow y = -\frac{1}{4} + 1 = \frac{3}{4}} (-\infty, 1]$

$\cup \Rightarrow \mathbb{R}$



$\xrightarrow{\text{در این قسمت}} \text{مقدار} \rightarrow y = |f(|x|) + 1| \geq ?$



$$a) y = \frac{m+1}{m-4} \quad R_f = R - \left\{ \frac{m}{f} \right\}$$

$$b) \sqrt{\frac{m+1}{m-4}} \rightarrow R - \left\{ \frac{m}{f} \right\}$$

$$y^2(m-4) = m+1 \rightarrow my^2 - 4y^2 = m+1 \\ my^2 - m = 4y^2 + 1 \rightarrow m(y^2 - 1) = 4y^2 + 1 \rightarrow m = \frac{4y^2 + 1}{y^2 - 1} \\ y^2 - 1 \neq 0 \rightarrow y \neq \pm 1 \rightarrow y \neq \pm 1 \rightarrow y > 0$$

$$2) y = \frac{m^2 + f}{\sqrt{m^2 + f}} \quad \frac{m^2 + f}{\sqrt{m^2 + f}} + \frac{1}{\sqrt{m^2 + f}} = \sqrt{m^2 + f} + \frac{1}{\sqrt{m^2 + f}} \xrightarrow{\text{نقطة الحد}} \sqrt{f} + \frac{1}{\sqrt{f}} \rightarrow \left[\frac{f}{\sqrt{f}}, +\infty \right)$$

$$3) = m^2 + \frac{1}{m^2 + f} \rightarrow m^2 = A \rightarrow y = A + \frac{1}{A + f} \rightarrow S(0) = \frac{1}{f} \rightarrow y - \frac{1}{2} = A + \frac{1}{A + f} - \frac{1}{f}$$

$$\frac{fA(A+f) + f - (A+f)}{f(A+f)} = \frac{fA^2 + fA - A}{f(A+f)} = \frac{fA^2 + fA - A}{f(A+f)} \xrightarrow{A > 0} \frac{A(fA + f - 1)}{f(A+f)} \xrightarrow{fA + f - 1 > 0} f(A+f) > 0$$

$$y - \frac{1}{2} > 0 \rightarrow y > \frac{1}{2} \rightarrow \left[\frac{1}{2}, +\infty \right) \\ y) \frac{m^2 + f}{m^2 - 1} \rightarrow m^2 - y = m^2 + f \rightarrow m^2 - m^2 = \varepsilon + y \rightarrow m^2 = \frac{y + \varepsilon}{y - 1} \rightarrow m = \pm \sqrt{\frac{y + \varepsilon}{y - 1}} \\ R_f = (-\infty, -\varepsilon] \cup (\varepsilon, +\infty) \\ \text{نقطة الحد} \rightarrow m = 1 \rightarrow y = 2 \\ \text{نقطة الحد} \rightarrow m = 0 \rightarrow y = -f \Rightarrow R_f = R - \left[-\varepsilon, 2 \right]$$

$$v) f(m) = \frac{-1 < A < 1}{\sin m + f} \rightarrow \frac{fA + \varepsilon}{A - f} \xrightarrow{yA - fy = fA + \varepsilon} yA - fA = fy + \varepsilon \rightarrow yA - fA = fy + \varepsilon \rightarrow A = \frac{fy + \varepsilon}{y - f} \rightarrow -1 \leq \frac{fy + \varepsilon}{y - f} \leq 1$$

$$\frac{fy + \varepsilon}{y - f} > -1 \rightarrow \frac{fy + \varepsilon + y - f}{y - f} > 0 \rightarrow \frac{\varepsilon y + f}{y - f} > 0 \rightarrow \frac{fy + 1}{y - f} > 0 \rightarrow \frac{-\frac{1}{f} - \frac{f}{1-f}}{+1-1} \\ \frac{fy + \varepsilon}{y - f} \leq 1 \rightarrow \frac{fy + \varepsilon - y + f}{y - f} \leq 0 \rightarrow \frac{fy + f}{y - f} \leq 0 \rightarrow \frac{y + f}{y - f} \leq 0 \rightarrow \frac{-\frac{f}{1-f}}{+1-1} \\ \Rightarrow [-\frac{f}{1-f}, -\frac{1}{f}]$$

$$h) \text{ ان } y = \left| \frac{m-1}{m} \right| \quad fm y = m - 1 \\ fm y - m = -1 \rightarrow m(\varepsilon y - 1) = -1 \rightarrow m = \frac{-1}{\varepsilon y - 1} \rightarrow \varepsilon y - 1 \neq 0 \rightarrow y \neq \frac{1}{\varepsilon} \\ y = R - \left\{ \frac{m}{f} \right\}$$

$$\text{نقطة الحد} \rightarrow [0, +\infty)$$

$$b) y = \sqrt{\frac{am - 1n}{-1 + fm}} \rightarrow \sqrt{\frac{a(m-f)}{f(m-f)}} \xrightarrow{m \neq 0} \sqrt{\frac{a}{f}} = \frac{r}{f} \rightarrow R = \left\{ \frac{r}{f} \right\}$$