

$$f(-x) - xf(x) = 2|x| + 1 \rightarrow f(-x) = xf(x) + 2|x| + 1 \quad -1$$

$$f(x) + xf(-x) = 2|-x| + 1 = 2|x| + 1$$

$$\rightarrow f(x) + x(xf(x) + 2|x| + 1) = 2|x| + 1$$

$$\rightarrow f(x) + x^2 f(x) + x(2|x| + 1) = 2|x| + 1 \rightarrow (1 + x^2)f(x) = (2|x| + 1)(1 - x)$$

$$\rightarrow f(x) = \frac{(2|x| + 1)(1 - x)}{(1 + x^2)}$$

$$f(x) + f(0) = x^2 - 2x + 4 \xrightarrow{x=0} 2f(0) = 4 \rightarrow f(0) = 2 \quad -2$$

$$\rightarrow f(x) = x^2 - 2x + 2 \rightarrow f(x^2) = x^4 - 2x^2 + 2$$

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2 \rightarrow x^2 + (1 - y)x + 2 + y = 0 \quad -3$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \Delta \geq 0 \rightarrow b^2 - 4ac \geq 0 \rightarrow (1 - y)^2 - 4(2 + y) \geq 0$$

$$\rightarrow y^2 - 4y - 7 \geq 0 \rightarrow (y - 7)(y + 1) \geq 0 \quad \frac{-1}{+\phi} - \frac{7}{\phi +} \Rightarrow R = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = 2x^2 - 5x + 1 \rightarrow x_5 = \frac{-b}{2a} = \frac{5}{4} \rightarrow y_5 = \frac{-17}{4}, a > 0 \Rightarrow R = \left[\frac{-17}{4}, +\infty\right) \quad -4$$

$$\text{ب) } y = \sqrt{-x^2 + 4x - 5} \rightarrow x_5 = 2 \rightarrow y_5 = 3, a < 0 \rightarrow R = \sqrt{(-\infty, 5]} = [0, 2]$$

$$\text{ج) } y = \sqrt{x^2 - 3x^2 + 5x + 1} \xrightarrow{\text{بزرگترین توان}} \text{عبارت فرد است} \rightarrow R = \sqrt{(-\infty, +\infty)} = [0, +\infty)$$

$$\text{د) } y = \log(x^3 - 4x^2 + 7x + 5) \rightarrow x^3 - 4x^2 + 7x + 5 > 0 \xrightarrow{\text{مربع به شرط عبارت}} R = [0, +\infty)$$

$$\Rightarrow R = \log[0, +\infty) = (-\infty, +\infty) = \mathbb{R}$$

$$ا) y = \frac{3x+1}{x-2} \rightarrow R = \mathbb{R} - \left\{ \frac{a}{c} \right\} \rightarrow R = \mathbb{R} - \left\{ \frac{3}{1} \right\} \quad -5$$

$$ب) y = \sqrt{\frac{x+1}{x-2}} \rightarrow R = \sqrt{\mathbb{R} - \{2\}} = [0, +\infty) - \{2\}$$

$$ج) y = \frac{x^2+4}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \rightarrow \sqrt{x^2+3} > 0, \min \sqrt{x^2+3} = \sqrt{3}$$

$$\rightarrow \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{3} \rightarrow R = \left[\frac{4\sqrt{3}}{3}, +\infty \right)$$

$$د) y = x^2 + \frac{1}{x^2+4} = x^2+4 + \frac{1}{x^2+4} - 4 \rightarrow x^2+4 > 0, \min(x^2+4) = 4$$

$$\rightarrow 4 + \frac{1}{4} = \frac{17}{4} \rightarrow \frac{17}{4} \leq x^2+4 + \frac{1}{x^2+4} \rightarrow \frac{1}{4} \leq x^2 + \frac{1}{x^2+4} \rightarrow R = \left[\frac{1}{4}, +\infty \right)$$

$$هـ) y = \frac{3x^2+4}{x^2-1} \rightarrow x = \pm \sqrt{\frac{y+4}{y-3}} \rightarrow \frac{-4}{+4} - \frac{3}{+4} \rightarrow R = (-\infty, -4] \cup (3, +\infty) \quad -6$$

مخرجی ترانه صفر نشود.

$$نکته: x^2 \in [0, +\infty) \rightarrow x=0 \rightarrow y = \frac{4}{-1} = -4$$

$$\rightarrow x^2-1=0 \rightarrow x^2=1 \checkmark$$

$$x=+\infty \rightarrow y=3$$

$$\rightarrow R = (-\infty, -4] \cup (3, +\infty)$$

$$f(x) = \frac{2\sin x + 4}{\sin x - 3} \quad \sin x \in [-1, 1] \rightarrow \sin x = -1 \rightarrow y = \frac{2}{-4} = -\frac{1}{2}$$

$$\sin x = 1 \rightarrow y = \frac{6}{-2} = -3$$

$$\sin x - 3 \neq 0 \rightarrow R = \left[-3, -\frac{1}{2} \right]$$

$$الف) y = \left| \frac{3x-1}{x} \right|$$

1- بجانب قائم = 0

بجانب افقي = $\frac{3}{x}$

$$R = [0, +\infty)$$

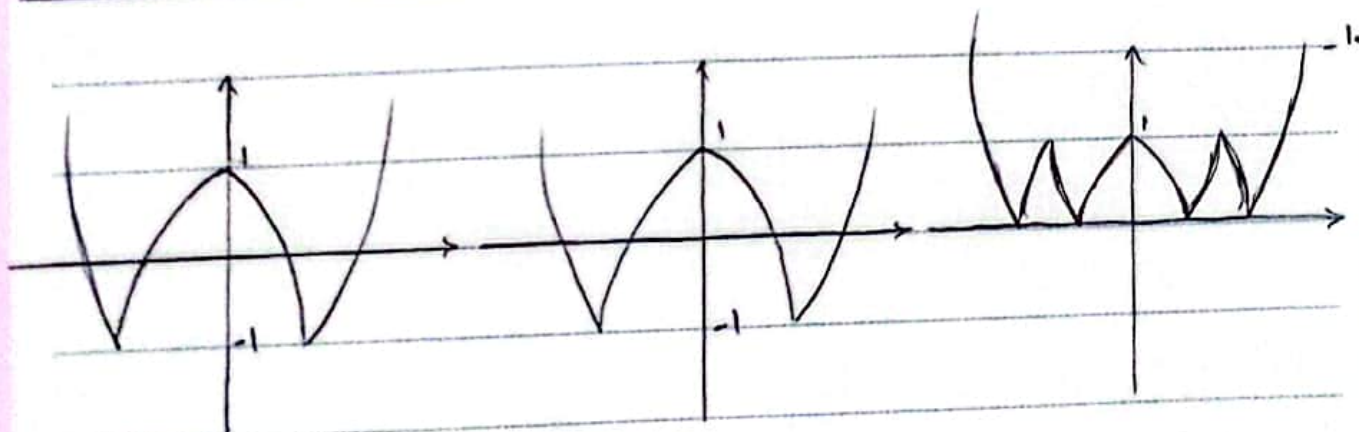
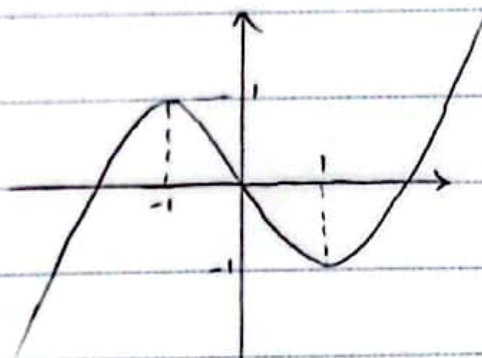
$$y = \frac{3}{x}$$

$$ب) y = \sqrt{\frac{9x-1}{x-1}} \rightarrow R = \sqrt{R - \left\{ \frac{9}{x} \right\}} = [0, +\infty) - \left\{ \frac{3}{x} \right\}$$

$$x \geq 0 \rightarrow x' - 2x = y \rightarrow y = (x-1)^2 - 1 \rightarrow y_{\min} = -1 \rightarrow R = [-1, +\infty)$$

$$x < 0 \rightarrow y = -x' - 2x \rightarrow y = -(x+1)^2 + 1 \rightarrow y_{\max} = 1 \rightarrow R = (-\infty, 1]$$

$$\rightarrow R_f = (-\infty, +\infty) = \mathbb{R}$$



$$y = f(x) + 1$$

$$y = f(|x|) + 1$$

$$y = |f(|x|) + 1|$$