

$$\text{a)} \quad y = \frac{|x|+1}{|x|-r}$$

(1) (1)

$$y' = \frac{x+1}{x-r} \rightarrow R_y = \mathbb{R} - \{1\} \rightarrow \frac{|x|+1}{|x|-r} \rightarrow y \rightarrow R_y = \mathbb{R} - \{1\}$$

$$\text{b)} \quad y = \frac{x^r+1}{|x|} \rightarrow \frac{|x|^r+1}{|x|} \rightarrow |x| + \frac{1}{|x|}$$

$$\frac{|x|+1}{a} \rightarrow a + \frac{1}{a} \quad a > 0 \rightarrow R_y = [-r, +\infty)$$

$$y = \frac{x^r+rx-r}{x+r} = \frac{(x+r)(x-1)+1}{x+r} = x-1 + \frac{1}{x+r} \rightarrow *$$

(P)

(r)

$$x^r+rx-r-1+1 = x^r+rx-r+1 = (x+r)(x-1)+1$$

$$* \text{ Jöbbsbl: } x-1 + \frac{1}{x+r} + r - r = x+r + \frac{1}{x+r} - r$$

$$\min a \Rightarrow x = -r \Rightarrow a + \frac{1}{a} = +r$$

$$a = -r+r = +1$$

$$\left. \begin{array}{l} \rightarrow x = -r \Rightarrow a = -r+r = -1 \Rightarrow a + \frac{1}{a} = -r \end{array} \right\} -r$$

$$R_y = (-\infty, -r] \cup [-r, +\infty)$$

$$f(x) = x^r - x + \frac{1}{x} = \frac{x^r - x + 1}{x} = \frac{(x - \frac{1}{r})^r}{(x - \frac{1}{r})^r + \frac{r}{x}} \rightarrow +\infty$$

(P)

(0,2)

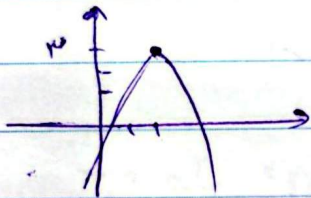
$$D_{f(x)} \Rightarrow (x - \frac{1}{r})^r + \frac{r}{x} = 0 \rightarrow (x - \frac{1}{r})^r = -\frac{r}{x} \quad D_f = \mathbb{R}$$

Spiegelung

$$(x - \frac{1}{r})^r = a \rightarrow f(x) = \frac{a}{a + \frac{r}{r}} \rightarrow R_f = \mathbb{R} \setminus \{r\}$$

ii) $y = \sqrt{-x^2 + rx - 1} = \sqrt{-(x-r)^2 + r^2} \Rightarrow \sqrt{(-\infty, r]} = [0, r] \text{ (F)}$

$$y = -x^2 + rx - 1 = r + r \Rightarrow -x^2 + rx - r + r = -(x-r)^2 + r^2 = -(x^2 - rx + r^2) + r^2$$



$$\leftarrow f(x) = -(x-r)^2 + r^2$$

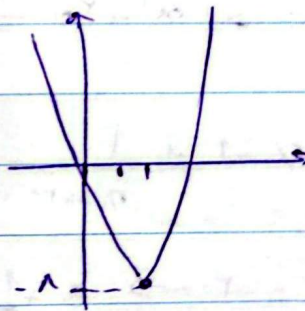
$$R_{f(x)} = \sqrt{(-\infty, r^2]}$$

$$R_f = [0, \sqrt{r}]$$

ii) $y = \sqrt{rx^2 - rx} = \sqrt{r(x^2 - x)} \Rightarrow R_y = [-\frac{1}{4}, +\infty)$

$$f(x) = rx^2 - rx$$

ext / $\frac{-b}{2a} = \frac{+1}{2} = \frac{1}{2}$



$$R_{f(x)} = [-\frac{1}{4}, +\infty)$$

iii) $\log(x^2 - x + 1)$

$$0 < x^2 - x + 1 \rightarrow R_{f(x)} = \mathbb{R} \checkmark$$

(2) (a)

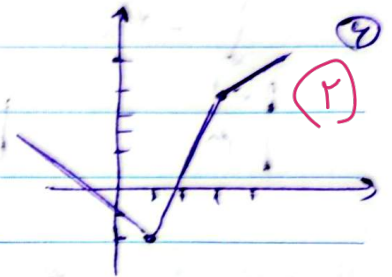
حل المسألة

$$\rightarrow y = \log_r \frac{m^2 + 1}{n} \rightarrow R_f = [1, +\infty)$$

$$a \leftarrow \frac{a^2 + 1}{n} \rightarrow x + \frac{1}{n} > 0 \xrightarrow{a > 0} \frac{a^2 + 1}{a} > 0 \rightarrow \log_r \frac{a^2 + 1}{a} > \log_r \frac{a^2 + 1}{a}$$

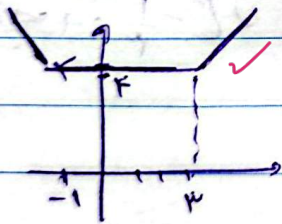
$$\text{الف) } y = |2x - 2| - |x - 3|$$

$$\begin{array}{c} \text{---} 1 \quad \quad \quad 3 \text{---} \\ | \quad \quad \quad | \\ -x - 1 \quad \quad \quad 2x - 2 - (-x + 3) \quad \quad \quad 2x - 2 - (x - 3) \\ \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ \quad \quad \quad 3x - 5 \quad \quad \quad 2x - x + 3 - 2 = x + 1 \\ \quad \quad \quad | \quad \quad \quad | \\ \quad \quad \quad -1 \quad \quad \quad 3 \end{array}$$



$$R_g = [-2, +\infty) \checkmark$$

$$\rightarrow y = |x - 3| + |x + 1|$$



مجموع كدانه:

$$\begin{array}{l} f(x) \left\{ \begin{array}{ll} x - 3 + x + 1 = 2x - 2 & x \leq -1 \rightarrow y \geq 4 \\ -x + 3 + x + 1 & -1 < x < 3 \rightarrow y = 4 \\ 2 - 2x & x \geq 3 \rightarrow y \geq 4 \end{array} \right. \end{array}$$

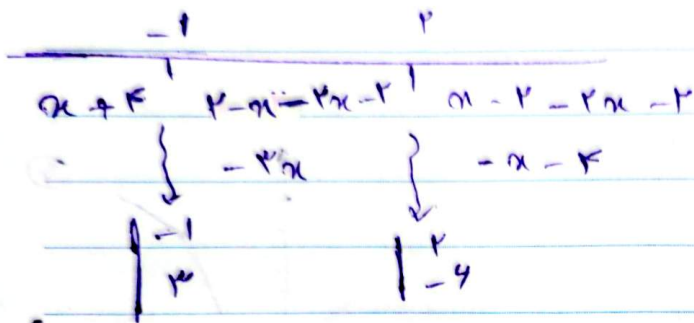
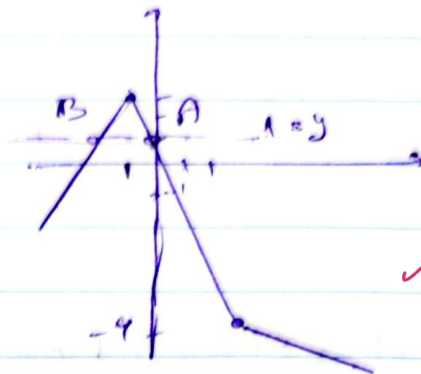
$$R_f = [4, +\infty) \checkmark$$

حلیہ پیرا

$$|x - r| = |y_n + r| \leq 1$$

(Y) (N)

$$y'(x-r) = 1 \quad (r_n + r)$$



$A \rightarrow xA \rightarrow y=1 \quad \left\{ \begin{array}{l} 1 = -x_n \rightarrow x_A = -\frac{1}{x} \\ f = -x_n \end{array} \right.$ A rögzítve

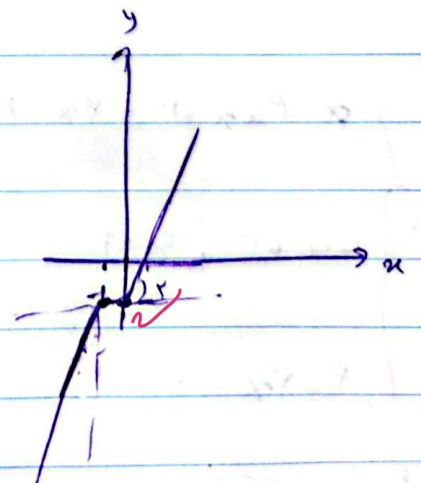
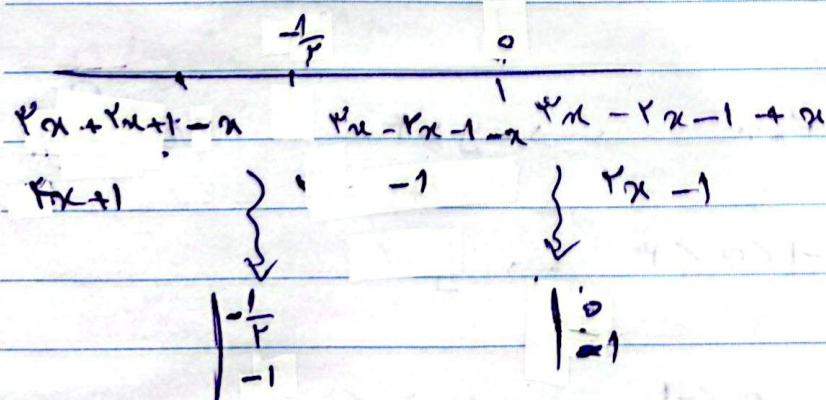
$$\alpha_B \rightarrow y=1 \quad \left\{ \begin{array}{l} 1 = \alpha + \kappa \\ y = \alpha + \kappa \end{array} \right. \rightarrow \alpha_B = -\kappa$$

$$(-\infty, -3] \cup [-\frac{1}{3}, +\infty) : -1$$

$$y = rx - |rx + 1| + |x|$$

①

②



$$R_4 = \mathbb{R} \checkmark$$

مسائل

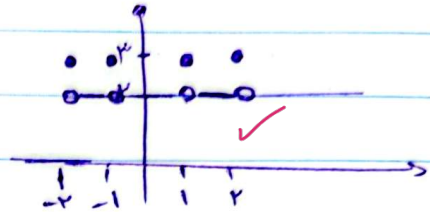
الف) $y = \lfloor n \rfloor + \lfloor 3 - n \rfloor$

$D_y = [-2, 2]$ (2) (9)

$y = \lfloor n \rfloor + \lfloor -n \rfloor + 3$

$\lfloor n \rfloor + \lfloor -n \rfloor = 0 \quad \leftarrow n \in \mathbb{Z} \rightarrow$

$\lfloor n \rfloor + \lfloor -n \rfloor = -1 \quad n \notin \mathbb{Z}$



$R_y = \{2, 3\}$

ب) $y = 3x - \lfloor x \rfloor$

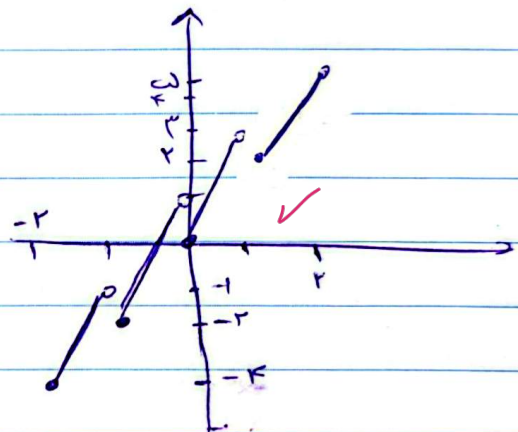
$y = 3x - 2 \quad x = 2 \rightarrow y = 4$

$y = 3x - 1 \quad 1 \leq x < 2$

$y = 3x \quad 0 \leq x \leq 1$

$y = 3x + 1 \quad -1 \leq x < 0$

$y = 3x + 2 \quad -2 \leq x < -1$



$R_y = [-2, 4]$

ج) $y = \sin^2 x - \sin x + 1 \rightarrow \min \boxed{\sin = -1} \rightarrow y = 3$ (10)

$R_f = [\frac{1}{4}, \frac{9}{4}]$

$\max \boxed{\sin = 1} \rightarrow y = 1$

$R_y = [\frac{1}{4}, \frac{9}{4}]$

$\frac{b}{4a} = +\frac{1}{4} \rightarrow \frac{1}{4} - \frac{1}{4} + 1 = -\frac{1}{4} + 1 = \frac{3}{4}$

د) $y = \sin^2 x + \sin^2 x + 1 \rightarrow \sin^2 x = t \rightarrow \min t = 0 \rightarrow 1$

$\max t = 1 \rightarrow 3$

$R_y = [1, 3]$ $R_y = [1, 3]$ $\frac{b}{4a} = -\frac{1}{4} = -\frac{1}{4}$

الف) $\square = |n| \rightarrow \begin{cases} |n| = 0 \rightarrow y = \frac{-1}{2} \\ |n| = +\infty \rightarrow y = 1 \end{cases} \xrightarrow[\text{مقدار}]{\text{مخرج میانه}} R_y = (-\infty, \frac{-1}{2}] \cup (1, +\infty)$

ب) $y = |n| + \frac{1}{|n|} \xrightarrow[\alpha + \frac{1}{\alpha} \geq 2]{\alpha > 0} |n| + \frac{1}{|n|} \geq 2 \rightarrow R_y = [2, +\infty)$

$$f(n) = \frac{(n - \frac{1}{2})^2}{(n - \frac{1}{2})^2 + \frac{3}{4}}$$

۳- مخرج کسر همواره مثبت است پس دامنه $\mathbb{R}$ است

طرفین در ضلع $\rightarrow y n^2 - y n + y = n^2 - n + \frac{1}{4} \rightarrow (y-1)n^2 - (y-1)n + y - \frac{1}{4} = 0$

$$n = \frac{y-1 \pm \sqrt{(y-1)^2 - 4(y-1)(y-\frac{1}{4})}}{2(y-1)} \rightarrow \Delta \geq 0 \rightarrow (y-1)(y-1-\cancel{4y+1}) \geq 0$$

$\frac{0}{-} \mid \frac{1}{+}$ $\Rightarrow y = [0, 1]$

غیرب n بنایه صفر شود یعنی $y \neq 1$ است

و تابع در ازای هیچ مقداری برابر یک نیست پس برد برابر است با: $R_y = [0, 1)$

می‌توانست از روش مربع هم استفاده کنیم:

$$f(n) = \frac{\square^2}{\square^2 + \frac{3}{4}}$$