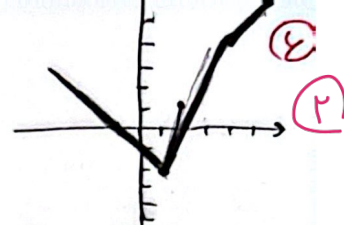


الف) $y = |x-2| - |x-3|$

$R_f = [-2, +\infty)$

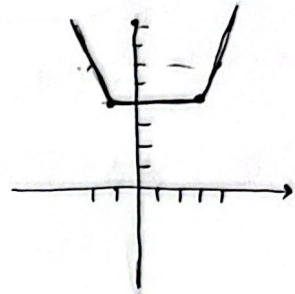
Handwritten piecewise analysis for $y = |x-2| - |x-3|$ with critical points at 2 and 3.



ب) $y = |x-2| + |x+1|$

Handwritten piecewise analysis for $y = |x-2| + |x+1|$ with critical points at -1 and 2.

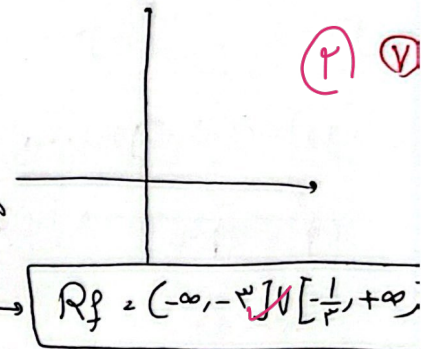
$R_f = [2, +\infty)$



$|x-2| - |x+2| \leq 1$

Handwritten piecewise analysis for the inequality $|x-2| - |x+2| \leq 1$.

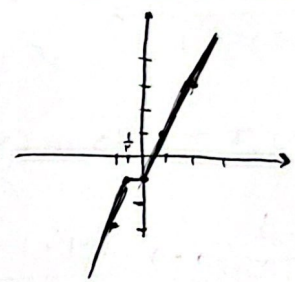
$(-\infty, -3] \cap [-\frac{1}{2}, +\infty) \cap \dots \rightarrow R_f = (-\infty, -3] \cup [-\frac{1}{2}, +\infty)$



$y = x - |x+1| + |x|$

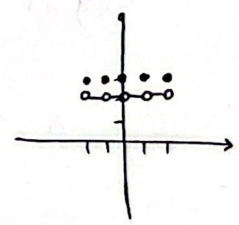
Handwritten piecewise analysis for $y = x - |x+1| + |x|$ with critical points at -1 and 0.

$R_f = \mathbb{R}$



الف) $y = [x] + [x-1]$

$R_f = \{1, 2\}$



ب) $y = x - [x]$

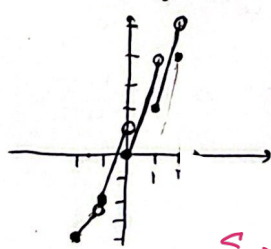
$(x, 1) \rightarrow x+2$

$[-1, 0) \rightarrow x+1$

$[0, 1) \rightarrow x$

$[1, 2) \rightarrow x-1$

$x \rightarrow x$



$R_y = [-1, 1)$

الف) $y = \sin^2 x - \sin^2 x + 1$

$\sin^2 x \in [0, 1]$

$\sin^2 x = \frac{1}{2} \rightarrow \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2}$

$R_f = [0, 1]$

ب) $\sin^2 x + \sin^2 x + 1$

$\sin^2 x \in [0, 1]$

$\sin^2 x \in [0, 1] \rightarrow \sin^2 x \in [-\frac{1}{2}, \frac{1}{2}]$

$R_f = [1, 3]$

$[\frac{1}{2}, 1]$

الف) $y = \frac{|n|+1}{|n|-2}$ $|n|-2y = |n|+1 \rightarrow |n|(y-1) = 1+2y \rightarrow |n| = \frac{1+2y}{y-1} \rightarrow \frac{1+2y}{y-1} \geq 0 \rightarrow \frac{-1}{y-1} \geq 0$ (1)

$R_f = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$ (2)

ب) $y = \frac{n^2+1}{|n|}$

$y = |n| + \frac{1}{|n|} \rightarrow R_f = [1, +\infty)$ (3)

$y = \frac{n^2+2n-2}{n+2}$ $n^2+2y = n^2+2n-2 \rightarrow n^2 + (2-y)n + (-2-2y) = 0 \rightarrow \Delta = (2-y)^2 - 4(-2-2y) \geq 0$ (2)

$(2-y)^2 - 4(-2-2y) \geq 0 \rightarrow y^2 + 4 - 4y + 8 + 8y \rightarrow y^2 + 12y + 12 \geq 0 \rightarrow (y+4)(y+3) \geq 0$

$\frac{-4}{+3} - \frac{-3}{+4} \rightarrow R_f = (-\infty, -4] \cup [-3, +\infty)$

$y = \frac{n^2-n+\frac{1}{\varepsilon}}{n^2-n+1}$ $\Delta = 1-4(1-\frac{1}{\varepsilon}) \leq 0 \rightarrow D_f = \mathbb{R}$ (1,5) (4)

$x \rightarrow n^2-n = t \rightarrow y = \frac{t+\frac{1}{\varepsilon}}{t+1}$

$yt+y = t+\frac{1}{\varepsilon} \rightarrow t(y-1) = \frac{1}{\varepsilon}-y \rightarrow t = \frac{\frac{1}{\varepsilon}-y}{y-1} \rightarrow \frac{\frac{1}{\varepsilon}-y}{y-1} \geq -\frac{1}{\varepsilon}$

$\frac{\frac{1}{\varepsilon} + \frac{\frac{1}{\varepsilon}-y}{y-1}}{y-1} \geq 0 \rightarrow \frac{y-1+1-y}{\varepsilon(y-1)} \geq 0 \rightarrow \frac{-y}{\varepsilon(y-1)} \geq 0 \rightarrow \frac{1}{-\frac{1}{\varepsilon} + \frac{1}{y-1}} \rightarrow [0, 1) \rightarrow R_f = [\frac{1}{\varepsilon}, 1)$

الف) $y = \sqrt{-n^2+5n-1}$ $\frac{\Delta}{4a} = \frac{-(12-5)}{-4} \rightarrow \frac{17}{4} = 4.25 \rightarrow (-\infty, 3] \xrightarrow{\text{نیرنگ}} [0, \sqrt{17}] \subset R_f$ (2) (5)

ب) $y = \sqrt[n]{\frac{n^2+1}{n}}$ $\frac{n^2+1}{n} > 0 \rightarrow n + \frac{1}{n} > 0 \rightarrow [1, +\infty) \rightarrow R_f = [1, +\infty)$ (2)

$n > 0 \leq n < 0$

الف) $y = \sqrt[n]{n^3-n+1n}$ $(-\infty, +\infty)$ (2) (5)

ب) $y = \sqrt[n]{2n^2-n}$ $-\frac{\Delta}{4a} = \frac{-4^2}{4} = -4 \rightarrow [-4, +\infty) \rightarrow R_f = [-4, +\infty)$

۳- مندرج کسر همواره مثبت است پس دامنه IR است

$$f(n) = \frac{(n - \frac{1}{r})^2}{(n - \frac{1}{r})^2 + \frac{r}{4}}$$

طرفین درمیان $\rightarrow yn^2 - yn + y = n^2 - n + \frac{1}{r} \rightarrow (y-1)n^2 - (y-1)n + y - \frac{1}{r} = 0$

$$n = \frac{y-1 \pm \sqrt{(y-1)^2 - 4(y-1)(y-\frac{1}{r})}}{2(y-1)} \rightarrow \Delta \geq 0 \rightarrow (y-1)(y-1-\cancel{4y}+1) \geq 0$$

$\Rightarrow y = [0, 1]$

مخبر n نباید منفرد شود یعنی $y \neq 1$ است

و تابع در انزای هیچ مقداری برابر یک نیست پس برد برابر است با: $R_y = [0, 1)$

میتوانستیم از روش مربع هم استفاده کنیم

$$f(n) = \frac{\square^2}{\square^2 + \frac{r}{4}}$$