

الف) $y = \frac{|x|+1}{|x|-2}$

(1)

$$y|x| - 2y = |x| + 1$$

$$y|x| - |x| = 2y + 1$$

$$|x|(y-1) = 2y+1 \rightarrow |x| = \frac{2y+1}{y-1}$$

$$\frac{-\frac{1}{2}}{+\frac{1}{2} \quad -\frac{1}{2}}$$

$$R_f = \left(\frac{-1}{2}\right] \cup (1, +\infty)$$

ب) $y = \frac{x^2+1}{|x|}$

$$|x| + \frac{1}{|x|} \xrightarrow{|x| \geq 0} R_f = [2, +\infty)$$

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$$y = \frac{x^2+2x-2}{x+3}$$

$$yx + 3y = x^2 + 2x - 2$$

(2)

$$0 = x^2 + 2x - yx - 2 - 3y$$

$$0 = x^2 + (2-y)x - 2-3y$$

$$\Delta \geq 0 \quad (2-y)^2 - 4(-2-3y) \geq 0$$

$$4 + y^2 - 4y + 8 + 12y \geq 0$$

$$y^2 + 8y + 12 \geq 0$$

$$(y+4)(y+2) \geq 0 \quad \frac{-4 \quad -2}{+\frac{1}{2} \quad -\frac{1}{2}}$$

$$R_f = (-\infty, -4] \cup [-2, +\infty)$$

(3)

$$f(x) = \frac{x^2 - x + \frac{1}{x}}{x^2 - x + 1}$$

$$x^2 - x + 1 \neq 0$$

$$\Delta = b^2 - 4ac = 1 - 4 = -3 < 0, a > 0$$

$$D_f = \mathbb{R} \leftarrow x^2 - x + 1 \text{ همیشه مثبت است} \leftarrow$$

$$yx^2 - yx + y = x^2 - x + \frac{1}{x}$$

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$$0 = x^2 - yx^2 - x + yx + \frac{1}{x} - y \rightarrow x^2(1-y) + x(y-1) + \frac{1}{x} - y = 0$$

$$a = \frac{y-1 \pm \sqrt{(y-1)^2 - r(y-1)(y-\frac{1}{r})}}{r(y-1)} \leadsto y \neq 1$$

$$(y-1)^2 - (y-1)(ry-1) \geq 0 \rightarrow (y-1)(y-1-ry+1) \geq 0$$

$$-ry(y-1) \geq 0$$

$$\rightarrow R_f = [0, 1)$$

$$\begin{array}{c} 0 \quad 1 \\ - \quad + \quad - \end{array}$$

(K)

$$\text{الف) } y = \sqrt{-x^2 + rx - 1} \quad \max \left| \begin{array}{l} \frac{-b}{2a} = \frac{-r}{-2} = \frac{r}{2} \\ -\frac{b^2}{4a} = -\frac{r^2 - 4}{4} = \frac{4-r^2}{4} \end{array} \right.$$

$$y = \sqrt{(-\infty, r]} \rightarrow R_f = [0, \sqrt{r}]$$

$$\text{ب) } y = \sqrt[3]{rx^3 - \lambda x} \quad \min \left| \begin{array}{l} \frac{-b}{2a} = \frac{\lambda}{r} = \frac{r}{r} \\ r \times r - 1 = (-1) \end{array} \right.$$

$$y = \sqrt[3]{[-1, +\infty)} \rightarrow R_f = [-r, +\infty)$$

$$\text{الف) } y = \log(a^x - x + 1)$$

$$\rightarrow (-\infty, +\infty)$$

$$\leadsto \log(0, +\infty) \rightarrow R_f = (-\infty, -\infty)$$

$$\log_1 0 = -\infty, \log_1^{+\infty} = +\infty \rightarrow \mathbb{R}$$

(5)

$$\text{ب) } y = \log_r \frac{2^{x+1}}{x} \rightarrow \frac{2^{x+1}}{x} > 0 \rightarrow x > 0$$

$$x + \frac{1}{x} \geq r \quad y = \log_r [r, +\infty) \rightarrow R_f = [1, +\infty)$$

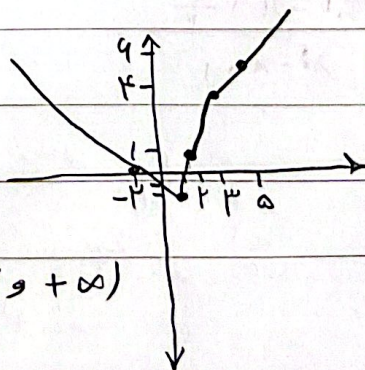
$$\text{الف) } y = |rx - r| - |x - r|$$

$$-x + r$$

$$\begin{array}{c|c|c} 1 & r & \\ \hline -rx + r + x - r = -x & rx - r & rx - r - x + r = x \end{array}$$

$$rx - r + x - r$$

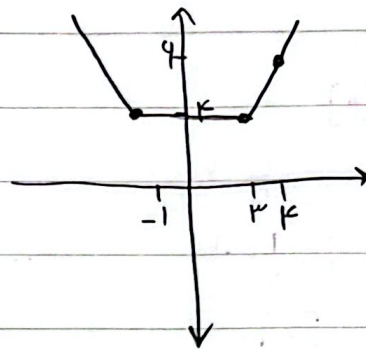
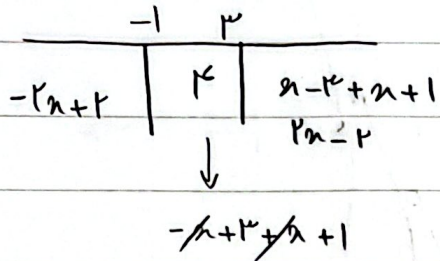
$$R_f = [-r, +\infty)$$



(4)

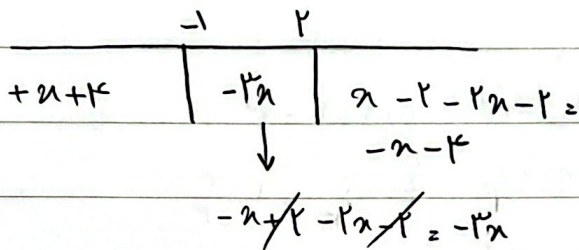
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ب) $y = |x-r| + |x+1|$

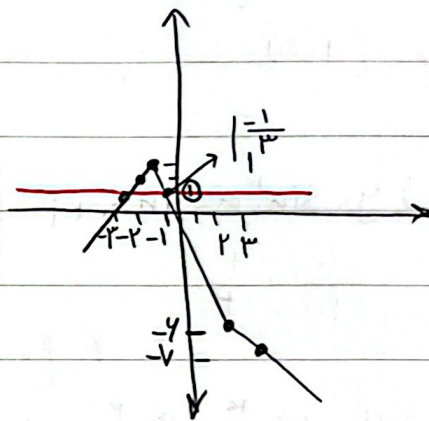


$R_f = [r, +\infty)$

١) $|x-r| - |x+1| \leq 1$

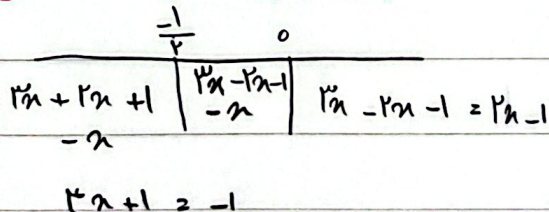


$R_f = \mathbb{R} - (-r, -\frac{1}{r})$

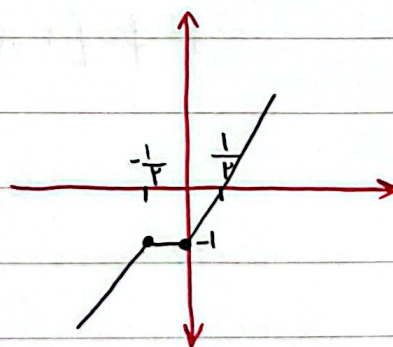


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$y = r|x| - |x+1| + |x|$



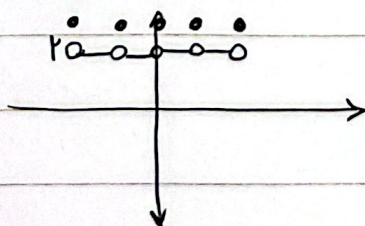
$R_f = \mathbb{R}$



٨

١) $y = [x] + [r-x] + [-x] + r$

ملاحظة: $y = [x] + [-x]$ راسه واحد به سمت بالا انتقال می دهیم



$$ب) y = \lfloor x \rfloor - [x]$$

$$-1 \leq x < -1 \rightarrow [x] = -1$$

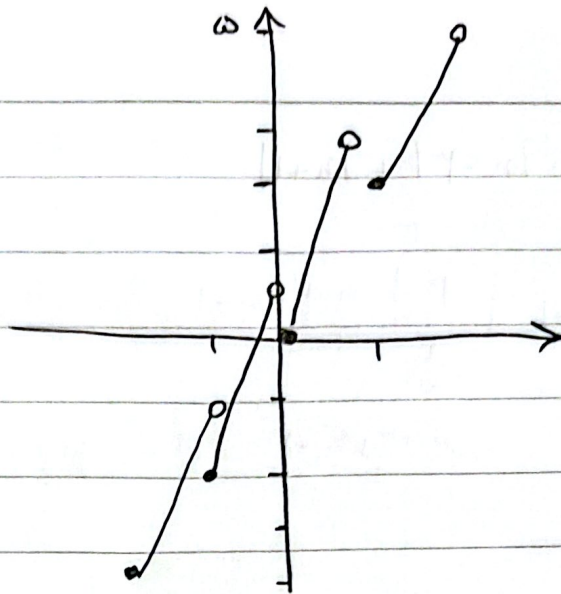
$$-1 \leq x < 0 \rightarrow [x] = -1$$

$$0 \leq x < 1 \rightarrow [x] = 0$$

$$1 \leq x < 2 \rightarrow [x] = 1$$

$$x = 2 \rightarrow y = 2$$

$$R_f = [-1, \infty)$$



$$الف) y = \sin^2 x - \sin x + 1$$

$$\frac{-b}{2a} = \frac{1}{2}$$

$$R_f = \left[\frac{1}{2}, 3 \right]$$

$$\sin^2 x = 1$$

$$1 - 1 + 1 = 1$$

$$\sin^2 x = -1$$

$$1 + 1 + 1 = 3$$

$$\sin^2 x = \frac{1}{2}$$

$$\frac{1}{2} - \frac{1}{2} + 1 = \frac{1}{2}$$

(10)

$$ب) y = \sin^2 x + \sin^2 x + 1$$

$$\frac{-b}{2a} = -\frac{1}{2}$$

$$R_f = [1, 3]$$

$$\sin^2 x = 0$$

$$0 + 0 + 1 = 1$$

$$\sin^2 x = 1$$

$$1 + 1 + 1 = 3$$

$$\sin^2 x = -\frac{1}{2}$$

x

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