

الف) $y = \frac{|x|+1}{|x|-2} \rightarrow |x|+1 = y|x|-2y \rightarrow 1+2y = y|x|-|x| \rightarrow 1+2y = |x|(y-1)$

(سوال ۱)

$\rightarrow \frac{1+2y}{y-1} = |x| \quad \frac{1}{1} \quad \frac{1}{-1} \quad R = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$

ب) $y = \frac{x^2+1}{|x|} \Rightarrow |x| + \frac{1}{|x|} = y \quad \frac{a+\frac{1}{a}}{a>0} \rightarrow R = [2, +\infty)$

$y = \frac{x^2+2x-2}{x+3} \rightarrow yx+3y = x^2+2x-2 \rightarrow x^2+(2-y)x-2-3y = 0$

(سوال ۲)

$x = \frac{y-2 \pm \sqrt{y^2-4y+4+12y}}{2} \rightarrow x = \frac{y-2 \pm \sqrt{y^2+8y+12}}{2} \Delta: y^2+8y+12 \geq 0 \quad \frac{-4}{1} \quad \frac{-2}{1} \quad y = -2, -4$

$R = (-\infty, -4] \cup [-2, +\infty)$

$f(x) = \frac{x^2-x+\frac{1}{x}}{x^2-x+1}$

(سوال ۳)

$\Delta < 0 \rightarrow$ ریشه ندارد $\rightarrow D_f = \mathbb{R}$

$y = \frac{x^2-x+\frac{1}{x}}{x^2-x+1} \rightarrow x^2-x+\frac{1}{x} = yx^2-yx+y \rightarrow (1-y)x^2+(y-1)x+\frac{1}{x}-y = 0$

$y \neq 1 \rightarrow x = \frac{1-y \pm \sqrt{-3y^2+3y}}{2(1-y)} \Delta: -3y^2+3y \geq 0 \rightarrow \frac{1}{-1} \quad \frac{1}{1} \rightarrow$

$y=1 \rightarrow \frac{-3}{1} = 0 \times$

$R = [0, 1]$

الف) $y = \sqrt{-x^2+4x+1} \rightarrow R = \sqrt{(-\infty, 3]} = [0, \sqrt{3}]$
 $x_{\max} = 2$
 $y_{\max} = 3$

(سوال ۴)

ب) $y = \sqrt[3]{4x^2-11x} \rightarrow R = \sqrt[3]{-1, +\infty} = [-2, +\infty)$
 $x_{\min} = 2$
 $y_{\min} = -1$

الف) $y = \log(\frac{x^3-x+1}{1}) = \log \mathbb{R} = \log(0+\infty) = (-\infty, +\infty)$

(سوال ۵)

ب) $y = \log_p \frac{x^2+1}{x} = \log_p^{(-\infty, -2] \cup [2, +\infty)} = \log_p^{[2, +\infty)} = [1, +\infty)$

$\frac{x^2+1}{x} = x + \frac{1}{x} \rightarrow (-\infty, -2] \cup [2, +\infty)$

الف) $y = |2x-2| - |x-3|$

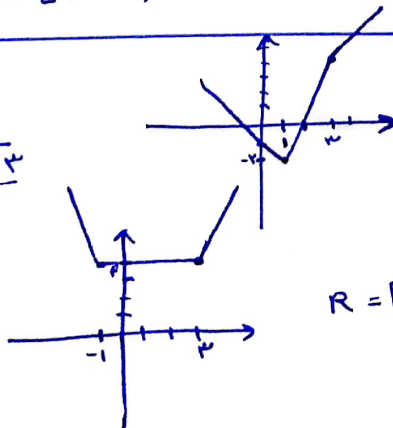
(سوال ۶)

$R = [2, +\infty)$

$\frac{x}{-2x+2+x-3} \quad \frac{x}{2x-2+x-3} \quad \frac{x}{x-2-x+3}$
 $\frac{-x}{-x-1} \quad \frac{x}{3x-5} \quad \frac{x}{x+1}$

ب) $y = |x-3| + |x+1|$

$R = [4, +\infty)$

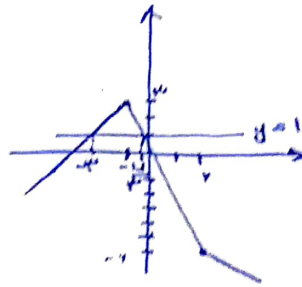


$$|x-2| - |2x+2| \leq 1$$

$$\frac{-x+2+2x+2}{x+2} - \frac{-x+2-2x-2}{-x-2} \leq 1$$

$$x+2 \leq 1 \rightarrow x \leq -1$$

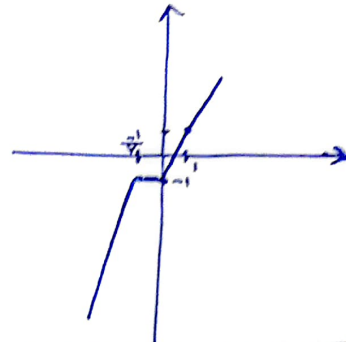
$$-x-2 \leq 1 \rightarrow x \geq -3$$



$$(-\infty, -3] \cup [-1, +\infty)$$

$$y = 3x - |2x+1| + |x|$$

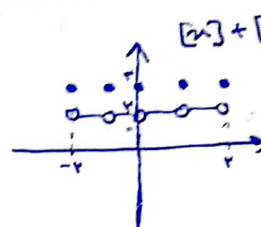
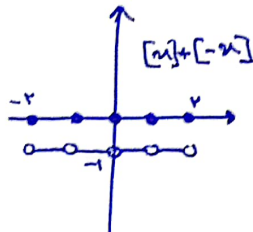
$$\frac{3x+2x+1-x}{x+1} - \frac{3x-2x-1-x}{-1} + \frac{3x-2x-1+x}{2x-1}$$



$$R_y = \mathbb{R}$$

مسئله (۸)

$$\cos(x) + \sin(x) + 3 = [x] + [-x] + 3$$

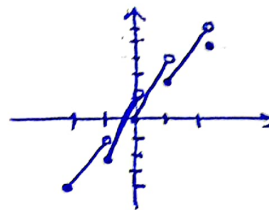


$$R = \{3\}$$

مسئله (۹)

$$y = 3x - [x]$$

$$\begin{aligned} [-2, -1) &\rightarrow 3x+2 \\ [0, 1) &\rightarrow 3x \\ [1, 2) &\rightarrow 3x-1 \\ \{2\} &\rightarrow 3x-2 \end{aligned}$$



$$R = [-2, 2]$$

مسئله (۱۰)

$$\sin^2 x - \sin x + 1$$

$$\sin x \begin{cases} 1 \\ -1 \end{cases} \rightarrow \begin{cases} 1 \\ 0 \end{cases}$$

$$R = \left[\frac{3}{4}, 1\right]$$

$$\sin^2 x + \sin^2 x + 1$$

$$\sin x \begin{cases} 1 \\ 0 \end{cases} \rightarrow \begin{cases} 1 \\ 0 \end{cases}$$

$$R = [1, 2]$$