

الف)  $y = \frac{|m+1|}{|m|-2}$

ب)  $y = \frac{n^2+1}{n} \Rightarrow y = \frac{|m|+1}{|m|} \xrightarrow{a=|m|} \frac{a+1}{a} \Rightarrow a > 0$

(2)

$|m| = -\frac{-y-1}{y-1} = \frac{y+1}{y-1} \geq 0$

$R_f: [2, +\infty)$

$y \mid \frac{1}{y-1} - \frac{1}{y} + \frac{1}{y} = 0 \Rightarrow R_f: (-\infty, -\frac{1}{2}] \cup (1, +\infty)$

$y = \frac{n^2+n-2}{n+2} \rightarrow y(n+2) = n^2+n-2 \rightarrow n^2+(2-y)n-2-2y=0$

(2)

$n = \frac{-b \pm \sqrt{\Delta}}{2a} \xrightarrow{\Delta \geq 0} \Delta = (2-y)^2 - 4(1)(-2-2y) \geq 0 \rightarrow 4 - 4y + y^2 + 8 + 16y \geq 0$

$y^2 + 12y + 12 \geq 0 \rightarrow (y+6)(y+2) \geq 0 \Rightarrow R_f: (-\infty, -6] \cup [-2, +\infty)$

$f(n) = \frac{n^2-n+1}{n^2-n+1} \neq 0$

$R_y = [0, 1) \Rightarrow \Delta = b^2 - 4ac = 1 - 4(1)(1) < 0 \Rightarrow D_f: \mathbb{R}$

(1, 75)

$f(n)n^2 - f(n)n + f(n) = n^2 - n + \frac{1}{2} \rightarrow (1 - f(n))n^2 + (-1 + f(n))n + (\frac{1}{2} - f(n)) = 0$

$R_f: [0, 1]$

الف)  $y = \sqrt{-n^2 + 2n - 1}$

ب)  $y = \sqrt{2n^2 - 4n}$

(2)

$a < 0 \rightarrow \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-2}{-2} = 1 \\ y_{\max} = -1 + 2 - 1 = 0 \end{cases}$

$a > 0 \rightarrow \begin{cases} x_{\min} = \frac{1}{2} = 1 \\ y_{\min} = 1 - 4 = -3 \end{cases}$

$R_f: (-\infty, 0] \rightarrow R_f: [0, 1]$

$R_f: [-3, +\infty) \rightarrow R_f: [-2, +\infty)$

الف)  $y = \log_r(n^2 - n + 1)$

ب)  $y = \log_r\left(\frac{n^2+1}{n}\right) > 0 \Rightarrow \log_r\left(n + \frac{1}{n}\right) > 0$

(2)

بدین ترتیب، چنانچه  $n$  از مرتبه  $1$  بزرگتر باشد

$(-\infty, -2] \cup [2, +\infty)$

$R_f: \mathbb{R}$

$\log_r \begin{cases} \log_r 1 \\ \log_r +\infty \end{cases} \rightarrow R_f: [1, +\infty)$

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                     |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|---------------------|----------|-----------|-------------|------------|-----------|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--------------------------------------------------|
| <p>الف) <math>y =  r^n - r  -  n - r </math></p> <table border="1"> <tr> <td><math>-r &lt; n &lt; r</math></td> <td><math>r^n - r</math></td> <td><math>r^n - r = n + r</math></td> </tr> <tr> <td><math>n &lt; -r</math></td> <td><math>r^n - r</math></td> <td><math>y = n - r</math></td> </tr> <tr> <td><math>n &gt; r</math></td> <td><math>r^n - r</math></td> <td><math>y = n + r</math></td> </tr> </table> <p><math>0 \rightarrow -1</math>   <math>-r</math>   <math>r</math>   <math>\infty</math></p> <p><math>R_f = [-r, +\infty)</math></p>                                                                                                          | $-r < n < r$                                                                                                                                                                                                                                                                                                                                                                                                                                               | $r^n - r$           | $r^n - r = n + r$   | $n < -r$ | $r^n - r$ | $y = n - r$ | $n > r$    | $r^n - r$ | $y = n + r$       | <p>ب) <math>y =  n - r  +  n + 1 </math>   <math>y \leq \frac{n+r}{n-1}</math></p> <p><math>n = -r \rightarrow y = 0</math>   <math>n = r \rightarrow y = 2r</math></p> <p>فقط <math>r = -r</math></p> <p><math>\Rightarrow R_f = [-1, +\infty)</math></p> |  |  |                                                  |
| $-r < n < r$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $r^n - r$                                                                                                                                                                                                                                                                                                                                                                                                                                                  | $r^n - r = n + r$   |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| $n < -r$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | $r^n - r$                                                                                                                                                                                                                                                                                                                                                                                                                                                  | $y = n - r$         |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| $n > r$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | $r^n - r$                                                                                                                                                                                                                                                                                                                                                                                                                                                  | $y = n + r$         |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| <p><math> n - r  -  r^n + r  \leq 1</math></p> <p><math> n - r  -  r^n + r  = 0</math></p> <table border="1"> <tr> <td><math>-r &lt; n &lt; r</math></td> <td><math>r^n - r</math></td> <td><math>r^n - r = n + r</math></td> </tr> <tr> <td><math>n &lt; -r</math></td> <td><math>r^n - r</math></td> <td><math>y = n - r</math></td> </tr> <tr> <td><math>n &gt; r</math></td> <td><math>r^n - r</math></td> <td><math>y = n + r</math></td> </tr> </table> <p><math>-r \rightarrow r</math></p> <p><math>R_f = I \cup II \cup III = (-\infty, -r] \cup [-\frac{1}{r}, +\infty)</math></p>                                                                       | $-r < n < r$                                                                                                                                                                                                                                                                                                                                                                                                                                               | $r^n - r$           | $r^n - r = n + r$   | $n < -r$ | $r^n - r$ | $y = n - r$ | $n > r$    | $r^n - r$ | $y = n + r$       | <p><math>-r^n \leq 1 \rightarrow r^n \geq -1</math></p> <p><math>n \geq -\frac{1}{r}</math></p>                                                                                                                                                            |  |  |                                                  |
| $-r < n < r$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $r^n - r$                                                                                                                                                                                                                                                                                                                                                                                                                                                  | $r^n - r = n + r$   |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| $n < -r$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | $r^n - r$                                                                                                                                                                                                                                                                                                                                                                                                                                                  | $y = n - r$         |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| $n > r$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | $r^n - r$                                                                                                                                                                                                                                                                                                                                                                                                                                                  | $y = n + r$         |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| <p><math>y = r^n -  r^n + 1  +  n </math></p> <table border="1"> <tr> <td><math>r^n + r^n + 1</math></td> <td><math>r^n - r^n + 1 - n</math></td> <td><math>r^n - r^n - 1 + n</math></td> </tr> <tr> <td><math>-n</math></td> <td><math>y = -1</math></td> <td><math>y = n - 1</math></td> </tr> <tr> <td><math>= 2n + 1</math></td> <td><math>-1</math></td> <td><math>1 \rightarrow 1</math></td> </tr> <tr> <td><math>-1 \rightarrow -r</math></td> <td></td> <td></td> </tr> </table> <p><math>\Rightarrow R_f = \mathbb{R}</math></p>                                                                                                                         | $r^n + r^n + 1$                                                                                                                                                                                                                                                                                                                                                                                                                                            | $r^n - r^n + 1 - n$ | $r^n - r^n - 1 + n$ | $-n$     | $y = -1$  | $y = n - 1$ | $= 2n + 1$ | $-1$      | $1 \rightarrow 1$ | $-1 \rightarrow -r$                                                                                                                                                                                                                                        |  |  | <p><math>\Rightarrow R_f = \mathbb{R}</math></p> |
| $r^n + r^n + 1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | $r^n - r^n + 1 - n$                                                                                                                                                                                                                                                                                                                                                                                                                                        | $r^n - r^n - 1 + n$ |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| $-n$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | $y = -1$                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $y = n - 1$         |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| $= 2n + 1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | $-1$                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $1 \rightarrow 1$   |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| $-1 \rightarrow -r$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                     |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| <p>الف) <math>y = [n] + [r - n] = [n] + [n] + r</math></p> <p><math>y = [n] + [n]</math></p> <p><math>R_f = \{r, r^2\}</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | <p>ب) <math>y = r^n - [n]</math></p> <p><math>-r \leq n &lt; -1</math>   <math>[n] = -r \rightarrow y = r^n + r</math></p> <p><math>-1 \leq n &lt; 0</math>   <math>[n] = -1 \rightarrow y = r^n + 1</math></p> <p><math>0 \leq n &lt; 1</math>   <math>[n] = 0 \rightarrow y = r^n</math></p> <p><math>1 \leq n &lt; r</math>   <math>[n] = 1 \rightarrow y = r^n - 1</math></p> <p><math>n = r</math>   <math>[n] = r \rightarrow y = r^n - r</math></p> |                     |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |
| <p>الف) <math>y = \sin^n n - \sin n + 1</math></p> <p>(min) <math>\sin n = -1 \rightarrow y = r</math></p> <p>(max) <math>\sin n = 1 \rightarrow y = 1</math></p> <p><math>R_f = [\frac{r}{r}, r]</math></p> <p><math>(-\frac{b}{ra})</math> <math>\sin n = \frac{1}{r} \rightarrow y = \frac{1}{2} - \frac{1}{r} + 1 = \frac{r}{r}</math></p> <p>ب) <math>y = \sin^n n + \sin^n n + 1</math></p> <p>(min) <math>\sin^n n = 0 \rightarrow y = 1</math></p> <p>(max) <math>\sin^n n = 1 \rightarrow y = r</math></p> <p><math>R_f = [1, r]</math></p> <p><math>(-\frac{b}{ra})</math> <math>\sin^n n = \frac{1}{r} \rightarrow \sin^n n \neq \frac{1}{r}</math></p> | <p><math>y = r^n - [n]</math></p> <p><math>-r \leq n \leq r</math></p> <p><math>R_f = [-r, r]</math></p>                                                                                                                                                                                                                                                                                                                                                   |                     |                     |          |           |             |            |           |                   |                                                                                                                                                                                                                                                            |  |  |                                                  |