

داریم و بنده A

الف) $y = \frac{|x|+1}{|x|-2}$

19, 18

حدا

$\lim_{x \rightarrow +\infty} y = 1$
 $\lim_{x \rightarrow -\infty} y = \frac{1}{2}$

بها و موصول

①

$R_f = (-\infty, \frac{1}{2}] \cup (1, +\infty)$

ب) $y = \frac{x^2+1}{|x|} = |x| + \frac{1}{|x|}$

②

$R_f = [2, +\infty)$

$y = \frac{x^2+x-2}{x+2}$

$yx + 2y = x^2 + x - 2$

③

$x(y-2) = x^2 - 2 - xy - 2$

$x = \frac{y-2 \pm \sqrt{(y-2)^2 + 4(xy+2)}}{2}$

$(y-2)^2 + 4(xy+2) \geq 0$

④

$y^2 - 4y + 4 + 4xy + 8 \geq 0$

$y^2 + 4y + 12 \geq 0 \rightarrow (y+2)(y+6) \geq 0$

$f(x) = \frac{x^2 - x + \frac{1}{x}}{x^2 - x + 1} = \frac{(x - \frac{1}{2})^2}{x^2 - x + 1}$

$R_f = (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty)$

⑤

$R_f = \mathbb{R}$

$R_g = [0, 1)$

$yx^2 - yx + y = x^2 - x + \frac{1}{x} - x(y-1) - x(y-1) + y - \frac{1}{x} = 0$

⑥

$x = \frac{y-1 \pm \sqrt{(y-1)^2 - 4(y-1)(y-\frac{1}{x})}}{2y-2}$

$(y-1)^2 - 4(y-1)(y-\frac{1}{x}) \geq 0$

$y^2 - 2y + 1 - 4(y^2 - \frac{y}{x} + \frac{1}{x}) \geq 0$

$y^2 - 2y + 1 - 4y^2 + \frac{4y}{x} - \frac{4}{x} \geq 0$

$-3y^2 + 2y - \frac{4}{x} + \frac{4y}{x} \geq 0 \rightarrow y^2(-y+1) \geq 0$

$-\frac{1}{x} + \frac{1}{x} = 0$

$f_{max} = \frac{\square^2}{\square^2 + \frac{1}{x}}$

$R_f = [0, 1]$

الف) $\sqrt{-x^2 + 4x - 1}$

$\max \left| \frac{-b}{2a} = \frac{-4}{-2} = 2 \right.$

⑦

$R_f = \sqrt{[-\infty, 3]} = [0, \sqrt{3}]$

ب) $y = \sqrt{x^2 - 12x}$

$\min \left| \frac{-b}{2a} = \frac{-12}{2} = -6 \right.$

$R_f = \sqrt{[-1, +\infty)}$

$R_f = [-2, +\infty)$

Arman

$$y = \log(x^2 - x + 1) \quad R_f = \mathbb{R} \checkmark$$

(r) (a)

$$y = \log_r \left(\frac{x^2 + 1}{x} \right) \rightarrow x + \frac{1}{x} \rightarrow (-\infty, -1] \cup [1, +\infty) \quad \left\{ \begin{array}{l} x + \frac{1}{x} = r \\ y = \log_r r = 1 \end{array} \right. \Rightarrow x + \frac{1}{x} = [1, +\infty)$$

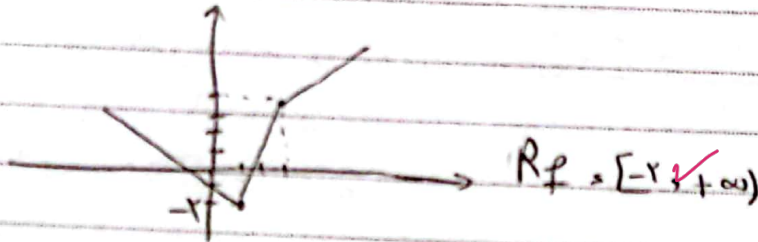
$$x + \frac{1}{x} = r \rightarrow y = \log_r r = 1$$

$$x \rightarrow \infty \rightarrow y = \log_r + \infty = +\infty \quad R_f = [1, +\infty)$$

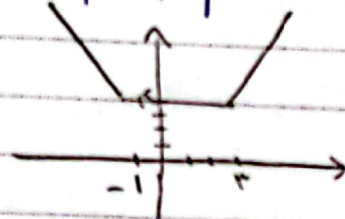
$$\text{c) } y = |x-2| - |x-4|$$

(r) (g)

$$\begin{array}{c} -x+2+x-4 = -x-2 \\ x-2+x-4 = x-6 \\ x-2-x+4 = x+2 \end{array}$$



$$\text{d) } y = |x-1| + |x+1|$$



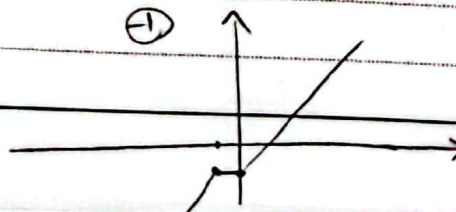
$$|x-2| - |x+2| \leq 1$$

$$\begin{array}{c} -x+2+x+2 = -x+4 \\ -x+2-x-2 = -x-2 \\ x-2-x-2 = -x-4 \end{array}$$

$$\begin{array}{l} x+2=1 \rightarrow x=-3 \\ -x-2=1 \rightarrow x=-3 \\ D = (-\infty, -3] \cup [-\frac{1}{2}, +\infty) \end{array}$$

$$y = x - |x+1| + |x|$$

$$\begin{array}{c} x+x+1-x = x+1 \\ x-x-1-x = -x-1 \\ x-x-1+x = x-1 \end{array}$$

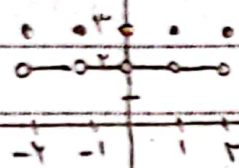


Arman

$$\text{الف) } y = [x] + [x-x] = [x] + [x] + x$$

$$x \in \mathbb{Z} \rightarrow y = 0 + x = x$$

$$x \notin \mathbb{Z} \rightarrow y = -1 + x = x-1$$



$$R_f = \{x\}$$

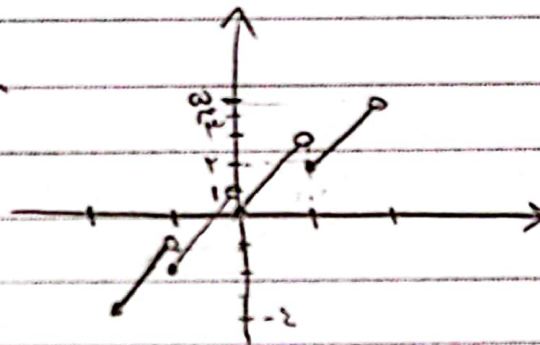
$$\text{ب) } y = x - [x] =$$

$$0 \leq x < 1 \rightarrow y = x - 0 = x$$

$$1 \leq x < 2 \rightarrow y = x - 1$$

$$-1 \leq x < 0 \rightarrow y = x + 1$$

$$-2 \leq x < -1 \rightarrow y = x + 2$$



$$R_f = [-x, 0)$$

$$\text{ج) } y = \sin^2 x - \sin x + 1$$

$$\sin x = -1, y = 1 + 1 + 1 = 3$$

$$\sin x = 1, y = 1 - 1 + 1 = 1$$

$$\sin x = \frac{1}{x}, y = \frac{1}{x} - \frac{1}{x} + 1 = \frac{x}{x}$$

$$R_f = [\frac{x}{x}, x]$$

$$\text{د) } y = \sin^2 x + \sin^2 x + 1$$

$$\sin^2 x = 1, y = 1 + 1 + 1 = 3$$

$$\sin^2 x = 0, y = 1$$

$$\sin^2 x = \frac{x}{x}$$

$$R_f = [1, x]$$