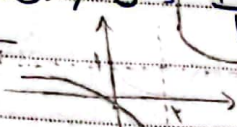


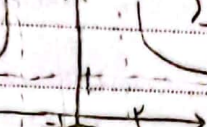
دارای دو نقطه

الف) $y = \frac{|x|+1}{|x|-2}$

$y = \frac{x+1}{x-2}$



$y = \frac{|x|+1}{|x|-2}$



$R_f = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$

ب) $y = \frac{x^2+1}{|x|} = |x| + \frac{1}{|x|}$

$R_f = [2, +\infty)$

$y = \frac{x^2+x-2}{x+2}$

$yx + 2y = x^2 + x - 2$

$x(y-2) = x^2 - 2 - 2y \Rightarrow x^2 - x(y-2) - 2y - 2 = 0$

$x = \frac{y-2 \pm \sqrt{(y-2)^2 + 4(2y+2)}}{2}$

$(y-2)^2 + 4(2y+2) \geq 0$

$y^2 - 2y + 4 + 8y + 8 \geq 0$

$y^2 + 6y + 12 \geq 0 \Rightarrow (y+3)(y+4) \geq 0$

$R_f = (-\infty, -4] \cup [-2, +\infty)$

$f(x) = \frac{x^2 - x + \frac{1}{x}}{x^2 - x + 1}$

$= \frac{(x - \frac{1}{2})^2}{x^2 - x + 1}$

$R_f = (-\infty, -\frac{1}{2}] \cup [-\frac{1}{2}, +\infty)$

$R_f = \mathbb{R}$

$yx^2 - yx + y = x^2 - x + \frac{1}{x} \Rightarrow x^2(y-1) - x(y-1) + y - \frac{1}{x} = 0$

$x = \frac{y-1 \pm \sqrt{(y-1)^2 - 4(y-1)(y-\frac{1}{x})}}{2(y-1)}$

$(y-1)^2 - 4(y-1)(y-\frac{1}{x}) \geq 0$

$y^2 - 2y + 1 - 4(y^2 - \frac{1}{x}y - y + \frac{1}{x}) \geq 0$

$y^2 - 2y + 1 - 4y^2 + \frac{4}{x}y + 4y - \frac{4}{x} \geq 0$

$-3y^2 + 6y - \frac{4}{x} \geq 0 \Rightarrow 3y(y-2) \geq \frac{4}{x}$

$\frac{1}{-1} + \frac{1}{1} = 0$

$R_f = [0, 1]$

الف) $\sqrt{-x^2 + 4x - 1}$

$\max \left| \frac{-b}{2a} = \frac{-2}{-2} = 1 \right|$

$R_f = \sqrt{[0, 4]} = [0, 2]$

ب) $y = \sqrt{x^2 - 12x}$

$\min \left| \frac{-b}{2a} = \frac{6}{1} = 6 \right|$

$R_f = \sqrt{[0, 36]} = [0, 6]$

$R_f = [-2, +\infty)$

Arman

$$y = \log(x^2 - x + 1) \quad R_f = \mathbb{R}$$

(2)

$$y = \log_r \left(\frac{x^2 + 1}{x} \right) \rightarrow x + \frac{1}{x} \rightarrow (-\infty, -1] \cup [1, +\infty) \quad \left\{ \begin{array}{l} x + \frac{1}{x} = r \\ y = \log_r r = 1 \end{array} \right. \Rightarrow x + \frac{1}{x} = [1, +\infty)$$

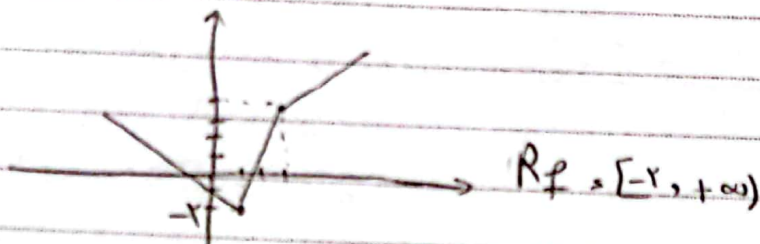
$$x + \frac{1}{x} = r \rightarrow y = \log_r r = 1$$

$$x \rightarrow \infty \rightarrow y = \log_r + \infty = +\infty \quad R_f = [1, +\infty)$$

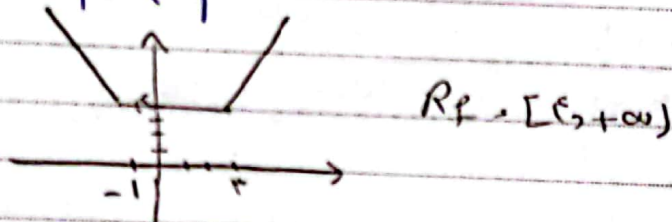
$$\text{c) } y = |x - 2| - |x - 1|$$

(3)

$$\begin{array}{c|c|c} -x+2+x-1 = -x+1 & x-x+2-1 = 1 & x-x-2+1 = -x-1 \end{array}$$



$$\text{d) } y = |x - 1| + |x + 1|$$



$$|x - 1| - |x + 1| \leq 1$$

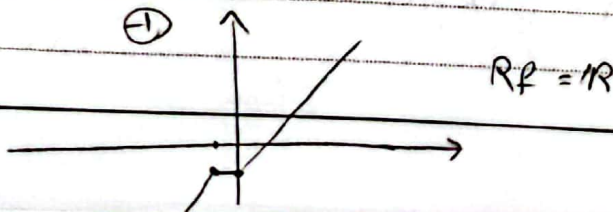
$$\begin{array}{c|c|c} -x+1+x+1 = 2 & -x+1-x-1 = -2 & x-1-x-1 = -2 \end{array}$$

$$\begin{aligned} x+1 &= 1 \rightarrow x = 0 \\ -x-1 &= 1 \rightarrow x = -2 \end{aligned}$$

$$D = (-\infty, -2] \cup [-1, +\infty)$$

$$y = x - |x + 1| + |x|$$

$$\begin{array}{c|c|c} x+x+1-x = x+1 & x-x-1-x = -x-1 & x-x-1+x = x-1 \end{array}$$



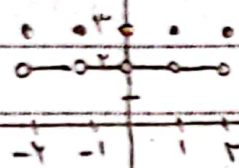
$$R_f = \mathbb{R}$$

Arman

الف) $y = [x] + [x-k] = [x] + [x] + k$

$x \in \mathbb{Z} \rightarrow y = 0 + k = k$

$x \notin \mathbb{Z} \rightarrow y = -1 + k = k-1$



$R_f = \{1, 2\}$

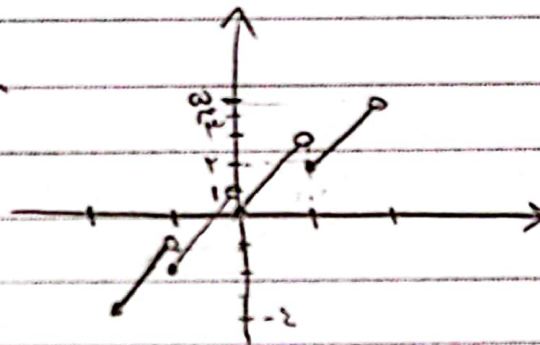
ب) $y = kx - [x]$

$0 \leq x < 1 \rightarrow y = kx - 0 = kx$

$1 \leq x < 2 \rightarrow y = kx - 1$

$-1 \leq x < 0 \rightarrow y = kx + 1$

$-2 \leq x < -1 \rightarrow y = kx + 2$



$R_f = [-\infty, \infty)$

و) $y = \sin^k x - \sin x + 1$

$\sin x = -1, y = 1 + 1 + 1 = 3$

$\sin x = 1, y = 1 - 1 + 1 = 1$

$\sin x = \frac{1}{k}, y = \frac{1}{k} - \frac{1}{k} + 1 = \frac{k}{k}$

$R_f = [\frac{k}{k}, 3]$

ب) $y = \sin^k x + \sin^r x + 1$

$\sin^k x = 1, y = 1 + 1 + 1 = 3$

$\sin^r x = 0, y = 1$

~~$\sin^k x = \frac{1}{k}, y = \frac{1}{k} + \frac{1}{k} + 1 = \frac{2k}{k} + 1$~~

$R_f = [1, 3]$