

الف) $0 \leq \sin^2(x + \frac{\pi}{4}) \leq 1 \rightarrow 0 \leq 2 \sin^2(x + \frac{\pi}{4}) \leq 2 \Rightarrow -1 \leq 2 \sin^2(x + \frac{\pi}{4}) - 1 \leq 1$
 $\Rightarrow R_f = [-1, 1]$

ب) $-1 \leq \cos^2 x \leq 1 \rightarrow -2 \leq 2 \cos^2 x \leq 2 \xrightarrow{-4} -4 \leq 2 \cos^2 x - 4 \leq -2 \Rightarrow R_g = [-2, -4]$

$x \in [-3, -2) \rightsquigarrow -1 - 3 = -4$

$x \in [-2, -1) \rightsquigarrow -1 - 2 = -3$

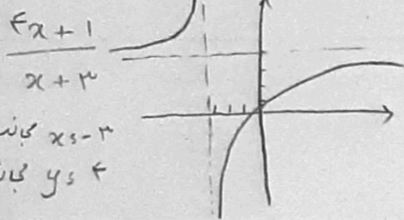
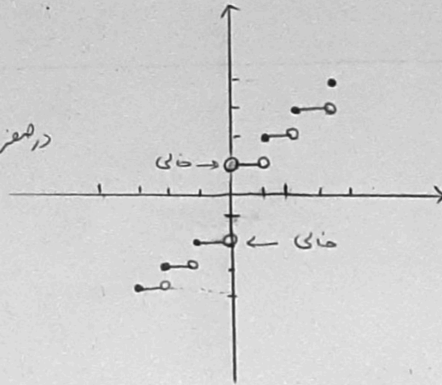
$x \in [-1, 0) \rightsquigarrow -1 - 1 = -2$ در قسمت (ن)

$x \in [0, 1) \rightsquigarrow 1 + 0 = 1$

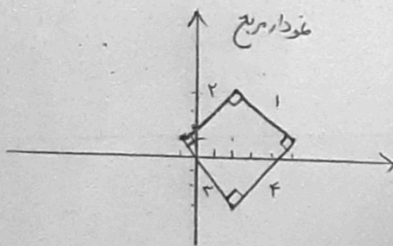
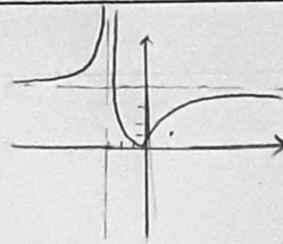
$x \in [1, 2) \rightsquigarrow 1 + 1 = 2$

$x \in [2, 3) \rightsquigarrow 1 + 2 = 3$

$x \in 3 \rightsquigarrow 1 + 3 = 4$



مردمطلق



ضلع ۱: $x - 2 + y - 1 = 3 \Rightarrow y \leq -x + 6 \quad (2 \leq x \leq 5)$

ضلع ۲: $-x + 2 + y - 1 = 3 \Rightarrow y \leq x + 2 \quad (-1 \leq x \leq 2)$

ضلع ۳: $-x + 2 - y + 1 = 3 \Rightarrow y \geq -x \quad (-1 \leq x \leq 2)$

ضلع ۴: $x - 2 - y + 1 = 3 \Rightarrow y \geq x - 4 \quad (2 \leq x \leq 5)$

الف) $x + \frac{1}{x} = a \rightarrow a \geq 2 \text{ یا } a \leq -2 \xrightarrow{\text{احتمال ندارد}} a \geq 2$
 $\rightarrow f(a), a + 2\sqrt{a} \xrightarrow{a \geq 2} \min f(a), 2 + 2\sqrt{2} \rightarrow D_f = (2 + 2\sqrt{2}, +\infty)$

ب) $-(\sin x - 2)(\sin x + 1) = -(\sin^2 x - 2 \sin x - 2) = -\sin^2 x + 2 \sin x + 2$

$\begin{cases} -1 \leq \sin x \leq 1 \\ -1 - 2 + 2 \leq 0 \\ -1 + 2 + 2 = 3 \end{cases} \Rightarrow R_f = [0, 3]$

$D_f = \{1, 2, 3, \dots, 9\} : f(1) = \frac{+29}{9}, f(2) = \frac{+24}{9}, f(3) = \frac{+19}{9}, \dots, f(9) = \frac{+1}{9}$

مجموع $\Rightarrow \frac{+(29 + 24 + \dots + 1)}{9} = +7$

$D_f \subset (-2, +\infty) \rightarrow$ تابع زیر را دنبال می‌کنیم است دانسته $\Rightarrow a \leq 0 \Rightarrow f(x) \leq \sqrt{bx+c}$

✓

$f(x) \leq 0 \Rightarrow bx+c \leq 0$
 $f(x) \leq 0 \Rightarrow bx+c=1$
 $\xrightarrow{\text{حل}} b \leq 1, c \leq -2 \Rightarrow g(x) \leq \sqrt{x^2+4} \xrightarrow{\text{بروزی}} [4, +\infty)$
 $\xrightarrow{\text{دارد}} [4, +\infty)$
 $\Rightarrow R_g \subset [2, +\infty)$

۸

$f(x) \leq \frac{x^2-6x}{x+2} \Rightarrow \frac{x(x-6)(x+2)}{(x+2)}$
 $\xrightarrow{\text{طبیعی}} f(x) \leq x^2-6x+1 \Rightarrow x^2-6x-1 \leq 0 \Rightarrow (x-3)(x+2) \leq 0$
 $\Rightarrow R_f \subset [-2, +\infty) - \{1\} \Rightarrow C \leq -1$
 $\begin{cases} x \leq 4 \rightarrow a \\ x \leq -2 \rightarrow b \end{cases}$
 محدودیت‌های دانسته

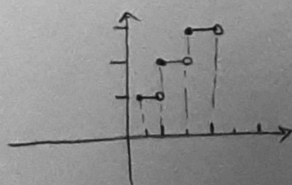
$f\left(\frac{a+b}{2}\right) \leq f\left(\frac{4-2}{-2}\right) = f(-1) = (-1)^2-6(-1)-1 = 1-2 \leq -1$ جواب

۹

تابع خطی است پس باید زیر را دنبال
 $(mx+n)^3$ این را می‌توانیم
 سه بار مشتق بگیریم
 $\sqrt[3]{(mx+n)^3} = \sqrt[3]{m^3x^3 + 3m^2nx^2 + 3mn^2x + n^3} = \sqrt{x^3+ax^2+bx-1}$
 $\Rightarrow n^3 = -1 \Rightarrow n = -1$
 $\Rightarrow m^3 = 1 \Rightarrow m = 1$
 $\Rightarrow x^3-4x^2+12x-1 \Rightarrow a = -4$
 $b \leq 12$
 $\Rightarrow D_f \subset [-4, 12]$
 $\Rightarrow R_f \subset [-1, 10]$ جواب
 $y = \sqrt[3]{(x-2)^3} = x-2$

۱۰

$[1, 4) \rightarrow 0+1 \leq 1$
 $[1, 3) \rightarrow 1+1 \leq 2$
 $[3, 4) \rightarrow 1+2 \leq 3$



۳ بار خط
 ۱/۳ طول