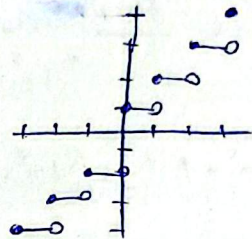


الف) $f(n) = 2 \sin^2(n + \frac{\pi}{2}) - 1$ $\begin{cases} \sin^2(n) \min: 0 \rightarrow y = -1 \\ \sin^2(n) \max: 1 \rightarrow y = 1 \end{cases} \rightarrow R: [-1, 1]$

ب) $g(n) = 3 \cos^2(n) - 4$ $\begin{cases} \cos^2(n) \min = 0 \rightarrow y = -4 \\ \cos^2(n) \max = 1 \rightarrow y = -1 \end{cases} \rightarrow R: [-4, -1]$

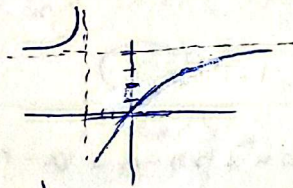
$y = \frac{n}{|n|} + [n] \rightarrow$

- $-n \leq n < -1 \rightarrow [n] = -n \rightarrow -1 - n = -4 = y$
- $-1 \leq n < 0 \rightarrow [n] = -1 \rightarrow -1 - 1 = -2 = y$
- $0 \leq n < 1 \rightarrow [n] = 0 \rightarrow -1 - 0 = -1 = y$
- $1 \leq n < 2 \rightarrow [n] = 1 \rightarrow -1 - 1 = -2 = y$
- $2 \leq n < 3 \rightarrow [n] = 2 \rightarrow -1 - 2 = -3 = y$
- $n = 0 \rightarrow [n] = 0 \rightarrow y = 1 + 0 = 1$

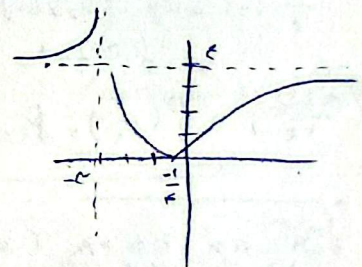


$y = \left| \frac{\varepsilon n + 1}{n + \varepsilon} \right| \rightarrow y = \frac{\varepsilon n + 1}{n + \varepsilon}$

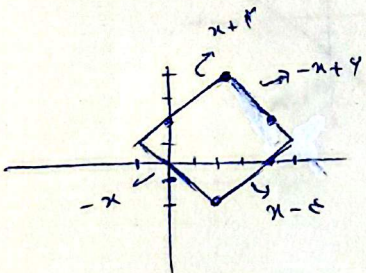
حداکثر = ε
حداقل = $-\varepsilon$



$y = \left| \frac{\varepsilon n + 1}{n + \varepsilon} \right|$



$|n-2| + |y-1| = 3 \rightarrow |y-1| = 3 - |n-2|$



$y-1 = 3 - |n-2| \rightarrow y = 4 - |n-2|$

$y-1 = |n-2| - 3 \rightarrow y = |n-2| - 2$

$D_f: 3 - |n-2| \geq 0 \rightarrow |n-2| \leq 3 \rightarrow -1 \leq n \leq 5$

$\begin{array}{c|c} 3 & 3 \\ \hline \varepsilon + n - 2 & 3 - n + 2 \\ = 2 + \varepsilon & = -n + 5 \end{array}$

$\begin{array}{c|c} 3 & 3 \\ \hline -n + 2 - 2 & n - 2 - 2 \\ = -n & = n - 4 \end{array}$

ج) $f(n) = n + \frac{1}{n} + 2\sqrt{n + \frac{1}{n}}$ $\rightarrow D_f: n + \frac{1}{n} \geq 0$

$\rightarrow n + \frac{1}{n} = t: t + 2\sqrt{t} \rightarrow t \min: \sqrt{2} \rightarrow y = 2 + 2\sqrt{2} \rightarrow R_f: [2 + 2\sqrt{2}, +\infty)$

$t = \frac{-b}{2a} = \frac{-1}{2} = -0.5$

د) $(3 - \sin n)(1 + \sin n) = -\sin^2 n + 2\sin n + 3$ $\begin{cases} \sin n \min = -1 \rightarrow y = -1 - 1 + 3 = 1 \\ \sin n \max = 1 \rightarrow y = -1 + 1 + 3 = 3 \end{cases} \rightarrow R_f: [1, 3]$

$$f(n) = -\frac{1}{2}n + 10 \rightarrow D_f: \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$\begin{aligned} \text{مثلاً} \rightarrow & \left(-\frac{1}{2}(1) + 10\right) + \left(-\frac{1}{2}(2) + 10\right) + \dots + \left(-\frac{1}{2}(9) + 10\right) = (9 \times 10) + (-12) = 9, -12 \\ \rightarrow & -\frac{1}{2} - \frac{2}{2} - \dots - \frac{9}{2} = -\frac{(1+2+\dots+9)}{2} = -\frac{9 \times 10}{2} = -12 \end{aligned}$$

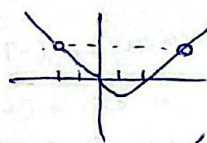
$$f(n) = \sqrt{an^2 + bn + c} \rightarrow D_f: [r, +\infty) \rightarrow a = 0$$

$$f(n) = \sqrt{bn + c} \quad f(n) = 1 \quad \sqrt{bn + c} = 1 \rightarrow bn + c = 1 \quad \frac{c - rb}{b} = 1 \quad b = 1 \quad c = -r$$

$$\rightarrow g(n) = \sqrt{n^2 + 2} \rightarrow R(n^2 + 2): [2, +\infty) \quad \sqrt{\quad} \quad [r, +\infty): \mathbb{R} \rightarrow \mathbb{R}$$

$$f(n) = \frac{n^2 - 2n}{n + 2} = \frac{n(n-2)(n+2)}{(n+2)} = n(n-2) = n^2 - 2n \quad n \neq -2 \rightarrow y = \lambda$$

$$-\frac{\Delta}{2a} = -\frac{2}{2} = -1 \rightarrow R_f: [-1, +\infty) \quad D_f: \mathbb{R} - \{a, b\} \rightarrow R_f: [c, +\infty)$$



از کتاب به یاد داشته باشید
صورت ناقص را به صورت کامل درآوریم

$$\begin{aligned} f(a) = f(b) = \lambda \quad \text{بزرگترین و کوچکترین} \\ n^2 - 2n = \lambda \rightarrow n^2 - 2n - \lambda = 0 \\ (n-2)(n+2) \rightarrow n \neq -2 \quad n \neq 2 \end{aligned}$$

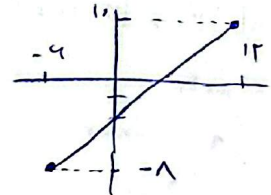
$$\rightarrow f\left(\frac{a+b}{2}\right) = f\left(\frac{-1+1}{2}\right) = f(0) = (-1)^2 - 2(-1) = 1 + 2 = 3$$

$$y = \sqrt[n]{n^2 + an^2 + bn - \lambda} \quad (1 \leq n \leq 2) \rightarrow n^2 + an^2 + bn - \lambda = (n-2)^2 = n^2 - 4n + 4 - \lambda$$

$$a = -4 \quad b = 12$$

$$\rightarrow y = \sqrt[n]{(n-2)^2} = n-2 \rightarrow D_f: [-4, 12]$$

$$\begin{aligned} f(-4) = -12 \\ f(12) = 10 \rightarrow R_f: [-12, 10] \end{aligned}$$

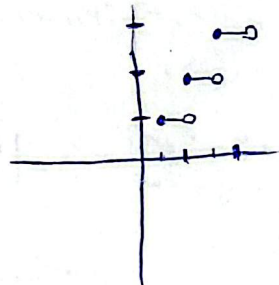


$$y = [n] + \left[n + \frac{1}{r}\right] \quad n \in \left[\frac{1}{r}, 2\right)$$

$$\frac{1}{r} \leq n < 1 \rightarrow \begin{aligned} [n] &= 0 \\ \left[n + \frac{1}{r}\right] &= 1 \end{aligned} \quad y = 1$$

$$1 \leq n < \frac{2}{r} \rightarrow \begin{aligned} [n] &= 1 \\ \left[n + \frac{1}{r}\right] &= 1 \end{aligned} \quad y = 2$$

$$\frac{2}{r} \leq n < 2 \rightarrow \begin{aligned} [n] &= 1 \\ \left[n + \frac{1}{r}\right] &= 2 \end{aligned} \quad y = 3$$



مثلاً (3)