

۲۰. افزایش:

الف) $f(x) = 2 \sin^2(x + \frac{\pi}{6}) - 1$

$\min \sin^2(t) = 0 \Rightarrow f(x) \leq -1$
 $\max \sin^2(t) = 1 \Rightarrow f(x) \leq 1$

$\Rightarrow R_f \subseteq [-1, 1]$

②

ب) $g(x) = 3 \cos^2 x - 4$

$\min \cos^2 x = 0 \Rightarrow -4 \leq g(x) \leq -1$
 $\max \cos^2 x = 1 \Rightarrow -1 \leq g(x) \leq 3$

$\Rightarrow R_g \subseteq [-4, 3]$

③

$y = \frac{x}{|x|} + [x], \quad [-3, 3] \quad D_y \subseteq \mathbb{R} - \{0\}$

$[-3, -2] \Rightarrow \frac{x}{|x|} + (-2) = -2$

$(0, 1) \Rightarrow 1$

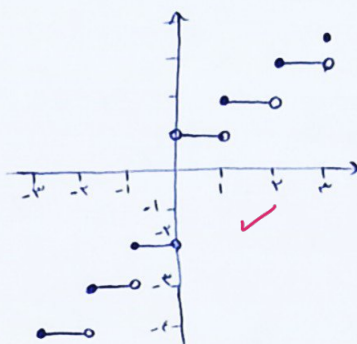
$[-2, -1] \Rightarrow \frac{x}{|x|} - 2 = -2$

$[1, 2] \Rightarrow 2$

$[2, 3] \Rightarrow 3$

$[-1, 0] \Rightarrow -2$

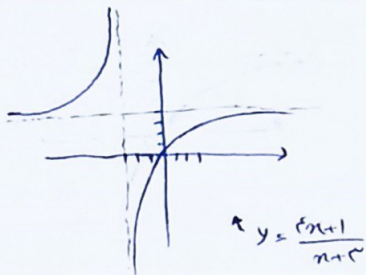
$\{3\} \Rightarrow 3$



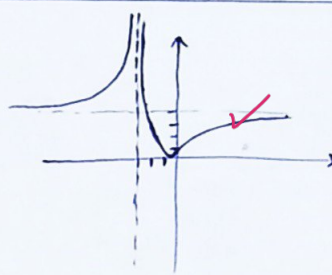
$y = \left| \frac{x+1}{x+3} \right|$

مخالفات $\Rightarrow x \leq -3$
 مخالفات $\Rightarrow y \leq 1$

$x \leq 0 \Rightarrow y \leq \frac{1}{3}$



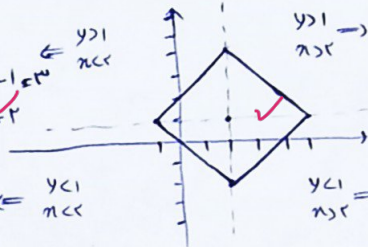
$y = \left| \frac{x+1}{x+3} \right|$



④

$|x-2| + |y-1| \leq 3$

$y > 1 \Rightarrow x-2+y-1 \leq 3 \Rightarrow y \leq x+6$
 $y < 1 \Rightarrow -x+2-y+1 \leq 3 \Rightarrow y \geq x-4$



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⑤

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \leq t^2 + 2t \Rightarrow R_f \subseteq [t+2\sqrt{t}, t+4\sqrt{t}]$

$\sqrt{x + \frac{1}{x}} \leq t$

$\max(t) \leq +\infty \Rightarrow +\infty$

$\min(t) \leq 1 \Rightarrow 1+2\sqrt{1}$

$x + \frac{1}{x} \xrightarrow{a \rightarrow 0} \frac{a+1}{a} \rightarrow [1, +\infty)$

$\frac{-b}{2a} = \frac{-1}{2} = -\frac{1}{2} \Rightarrow x$

ب) $y = (1 - \sin x)(1 + \sin x)$

$\sin x \leq 1$

$\sin x \geq -1$

$\max(\sin x) = 1 \Rightarrow 0$

$\min(\sin x) = -1 \Rightarrow 0$

$\Rightarrow R_y \subseteq [0, 1]$

$\frac{m_1 + m_2}{r} \leq x \leq \frac{-b}{ra} \leq \frac{r-1}{r} \leq 1$

$\frac{-b}{ra} \leq 1 \Rightarrow 1$

⑥

برساختن تابع - تکلیف ۵

$$f(n) \in \frac{1}{n^2} n + 10 \Rightarrow D_f \subseteq \{1, 2, \dots, 9\}$$

$$\left(-\frac{1}{n^2}(1)+10\right) + \left(-\frac{1}{n^2}(2)+10\right) + \dots + \left(-\frac{1}{n^2}(9)+10\right) = \frac{(9 \times 10)}{n^2} + \frac{1}{n^2} (1+2+\dots+9) \leq 90-10 \leq \frac{9 \times 10}{n^2} \leq 0$$

$$f(n) = \sqrt{x^2 + bx + c}, D_f \subseteq [2, +\infty), f(1) \leq 1$$

$$g(n) = \sqrt{bn^2 + an - 2c} \leq \sqrt{n^2 + 1}$$

در حالتی که عبارت درجه ۲
به ریشه باشد علامت
آن در دو طرف ریشه یکسان است پس این عبارت
درجه ۲ مثبت و (a > 0)

$$\Rightarrow \begin{cases} 2b+c=0 \\ 3b+c=1 \end{cases} \Rightarrow \begin{cases} b=-1 \\ c=2 \end{cases}$$

$$[2, +\infty) \xrightarrow{f} [R_g \subseteq [2, +\infty)] \leftarrow \text{ext} \mid \begin{cases} x \leq \frac{b}{a} \leq 0 \\ y \leq 1 \end{cases}$$

$$f(n) \leq \frac{n^2 - 2n}{n+2}, D_f \subseteq \mathbb{R} - \{a, b\}, R_f \subseteq [c, +\infty) - \{1\}$$

$$\frac{n(n-2)}{n+2} \leq \frac{n(n+2)(n-2)}{n+2} \Rightarrow n(n-2) \leq n^2 - 2n - 1 \leq 0 \Rightarrow \begin{cases} n \leq -1 \\ n \geq 3 \end{cases} \Rightarrow a, b \Rightarrow (a+b \leq 2)$$

$$\hookrightarrow n^2 - 2n = y \rightarrow \text{ext} \mid \begin{cases} n \leq 1 \\ y \leq -1 \end{cases} \xrightarrow{a, b} [-1, +\infty) \Rightarrow (c \leq 1)$$

$$f\left(\frac{a+b}{2}\right) = f\left(\frac{1}{2}\right) \leq f(-1) \leq 1$$

$$y \leq \sqrt{x^3 + ax^2 + bx - 1}, D_y \subseteq [a, b]$$

$$\text{ضبطی } y \leq ax + b \Rightarrow \begin{cases} \text{صورت ضرب در زیر رادیکال} \\ \text{یک است و عدد ثابت آن برابر با (1) است پس} \\ \text{تابع همصورت (x-2) است} \end{cases} \Rightarrow (x-2)^3 \leq x^3 - 4x^2 + 12x - 8 \leq x^3 + ax^2 + bx - 1 \Rightarrow \begin{cases} a \leq -2 \\ b \leq 12 \end{cases}$$

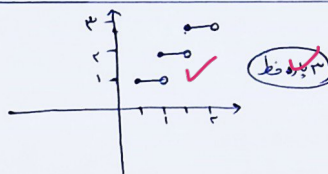
$$y \leq x-2 \Rightarrow \begin{cases} -4 \leq x \leq 12 \\ -1 \leq x-2 \leq 10 \end{cases} \Rightarrow R_y \subseteq [-1, 10]$$

$$y \in [n] + \left[n + \frac{1}{n}\right] \text{ و } \left[\frac{1}{n}\right] \text{ و } [2]$$

$$\left[\frac{1}{n}\right] \text{ و } [2] \rightarrow 0+1 \leq 1$$

$$\left[1 + \frac{1}{n}\right] \rightarrow 1+1 \leq 2$$

$$\left[\frac{2}{n}\right] \rightarrow 1+2 \leq 3$$



(۲) (۱۰)