

الف) $f(x) = 2 \sin^2(x + \frac{\pi}{2}) - 1$

$\min \sin^2(t) = 0 \Rightarrow f(x) = -1$
 $\max \sin^2(t) = 1 \Rightarrow f(x) = 1$

$\Rightarrow R_f \subseteq [-1, 1]$

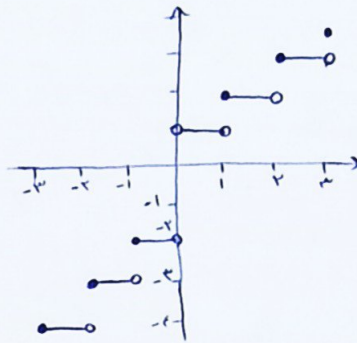
ب) $g(x) = 3 \cos^2 x - 7$

$\min \cos^2 x = 0 \Rightarrow -7 \leq g(x) \leq -4$
 $\max \cos^2 x = 1 \Rightarrow -4 \leq g(x) \leq -1$

$\Rightarrow R_g \subseteq [-7, -1]$

$y = \frac{\pi}{12} + [x], [-3, 3]$ $D_y \subseteq \mathbb{R} - \{0\}$

$[-3, -2] \Rightarrow \frac{\pi}{12} + (-2) = -\frac{23\pi}{12}$ $(0, 1) \subseteq 1$
 $[-2, -1] \Rightarrow \frac{\pi}{12} - 2 = -\frac{23\pi}{12}$ $[1, 2] \subseteq 2$
 $[-1, 0] \Rightarrow -2$ $[2, 3] \subseteq 3$
 $\{3\} \Rightarrow 4$

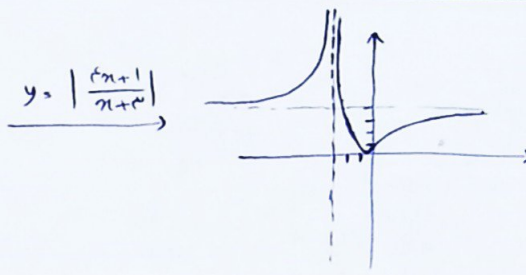


$y = \left| \frac{x+1}{x+3} \right|$

$x \leq -3 \Rightarrow$ مخالفاتی
 $x \geq -3 \Rightarrow$ یکنواختی

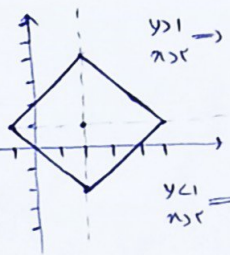
$x = 0 \Rightarrow y = \frac{1}{3}$

$y = \frac{x+1}{x+3}$



$|x-2| + |y-1| \leq 3$

$x < 2, y < 1 \Rightarrow -x+2+y-1 \leq 3 \Rightarrow y \leq x+2$
 $x > 2, y < 1 \Rightarrow x-2+y-1 \leq 3 \Rightarrow y \leq -x+6$
 $x < 2, y > 1 \Rightarrow -x+2-y+1 \leq 3 \Rightarrow y \geq x-2$
 $x > 2, y > 1 \Rightarrow x-2-y+1 \leq 3 \Rightarrow y \geq -x+6$



الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \leq t^2 + 2t \Rightarrow R_f \subseteq [t + 2\sqrt{t}, +\infty)$

$\sqrt{x + \frac{1}{x}} \leq t$

$x + \frac{1}{x} \leq t^2$ $\frac{a+1}{a} \geq \frac{b+1}{b} \Rightarrow \frac{a-b}{ab} \geq 0$

$\frac{a-b}{ab} \geq 0 \Rightarrow \frac{a-b}{a} \geq 0 \Rightarrow a \geq b$

ب) $y = (2 - \sin x)(1 + \sin x)$

$\max \sin x = 1 \Rightarrow y = 1$
 $\min \sin x = -1 \Rightarrow y = 0$

$\Rightarrow R_y \subseteq [0, 1]$

$\frac{a+b}{c} \leq \frac{a}{c} \leq \frac{b}{c} \Rightarrow \frac{a-b}{c} \leq 0$

برساختن تابع - تکلیف ۵

$$f(n) \leq \frac{1}{n^2} n + 10 \Rightarrow D_f \subseteq \{1, 2, \dots, 9\}$$

$$\left(-\frac{1}{n^2}(1)+10\right) + \left(-\frac{1}{n^2}(2)+10\right) + \dots + \left(-\frac{1}{n^2}(9)+10\right) = \frac{(9 \times 10)}{n^2} + \frac{1}{n^2} (1+2+\dots+9) \leq 90-10 \leq (VU)$$

$$\frac{9 \times 10}{n^2} \leq 80$$

$$f(n) = \sqrt{n^2 + bn + c}, D_f \subseteq [r, +\infty), f(r) \leq 1$$

$$g(n) = \sqrt{bn^2 + an - 2c} \leq \sqrt{n^2 + r}$$

در حالتی که عبارت درجه ۲
به ریشه باشد علامت $\Rightarrow$
آن در دو طرف ریشه یکسان است پس این عبارت
درجه ۲ نبوده و (a=0)

$$\begin{aligned} & \Rightarrow \begin{cases} rb+c=0 \\ rb+c \leq 1 \\ (b=1), (c=-1) \end{cases} \end{aligned}$$

$$[r, +\infty) \xrightarrow{f} [Rg \subseteq [r, +\infty)] \leftarrow \text{ext} \mid \begin{cases} x \leq \frac{b}{ra} \leq 0 \\ y \leq r \end{cases}$$

$$f(n) \leq \frac{n^2 - 2n}{n+2}, D_f \subseteq \mathbb{R} - \{a, b\}, R_f \subseteq [c, +\infty) - \{1\}$$

$$\frac{n(n^2 - 2)}{n+2} \leq \frac{n(n+2)(n-2)}{n+2} \Rightarrow n(n-2) \leq 1 \Rightarrow \begin{cases} n^2 - 2n - 1 \leq 0 \\ n^2 - 2n - 1 \geq 0 \end{cases} \Rightarrow \begin{cases} n \leq -1 \\ n \geq 3 \end{cases} \Rightarrow a, b \Rightarrow (a+b \leq 2)$$

$$\hookrightarrow n^2 - 2n = y \rightarrow \text{ext} \mid \begin{cases} n \leq 1 \\ y \leq -1 \end{cases} \xrightarrow{a, b} [-1, +\infty) \Rightarrow (c \leq -1)$$

$$f\left(\frac{a+b}{rc}\right) = f\left(\frac{r}{-1}\right) \leq f(-1) \leq 1$$

$$y \leq \sqrt{x^3 + ax^2 + bx - 1}, D_y \subseteq [a, b]$$

فرض $y \leq ax + b$
چون ضریب x^3 در زیر رادیکال
یک است و عدد ثابت آن برابر با (۱) است پس
تابع y صعودی است $(x-2)^3$ است

$$(x-2)^3 \leq x^3 - 4x^2 + 12x - 8 \leq x^3 + ax^2 + bx - 1 \Rightarrow \begin{cases} a \leq -3 \\ b \leq 12 \end{cases}$$

$$y \leq x-2 \quad -4 \leq x \leq 12 \Rightarrow R_y \subseteq [-4, 12]$$

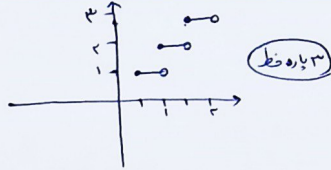
برای جابجایی هر دو تابع ابتدا آن را
ساخته و گوییم

$$y \leq [x] + \left[n + \frac{1}{r}\right] \text{ و } \left[\frac{1}{r}, 2\right]$$

$$\left[\frac{1}{r}, 1\right] \rightarrow 0+1 \leq 1$$

$$\left[1, \frac{2}{r}\right] \rightarrow 1+1 \leq 2$$

$$\left[\frac{2}{r}, 2\right] \rightarrow 1+2 \leq 3$$



(۱۰)