

الف) $f(n) = r \sin^2(n + \frac{\pi}{2}) - 1$
 $\sin^2(n + \frac{\pi}{2})$
 $\min = 0 \rightarrow f(n) = -1$
 $\sin^2(n + \frac{\pi}{2})$
 $\max = 1 \rightarrow f(n) = 1$

$R_f: [-1, 1]$

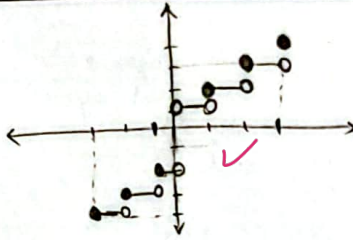
ب) $g(n) = r \cos^2 n - 2$
 $\cos^2 n \min = 0 \rightarrow g(n) = -2$
 $\cos^2 n \max = 1 \rightarrow g(n) = -1$

$R_g: [-2, -1]$

$y = \frac{\pi}{n} + [n]$

$-1 + [n] \mid 1 + [n]$

$-2 \leq n < -1 \rightarrow y = -1 + (-2) = -3$
 $-1 \leq n < 0 \rightarrow y = -1 + (-1) = -2$

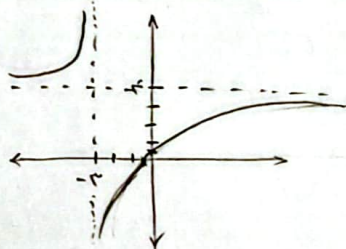


$0 \leq n < 1 \rightarrow y = 1 + 0 = 1$
 $1 \leq n < 2 \rightarrow y = 1 + 1 = 2$
 $2 \leq n < 3 \rightarrow y = 1 + 2 = 3$
 $n = 3 \rightarrow y = 1 + 3 = 4$

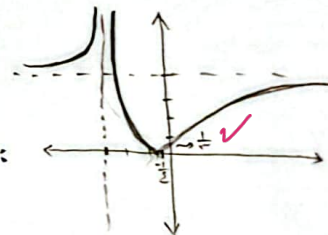
$y = \left| \frac{n+1}{n+2} \right|$

$\lim_{n \rightarrow -2} y = \frac{a}{b} = r$

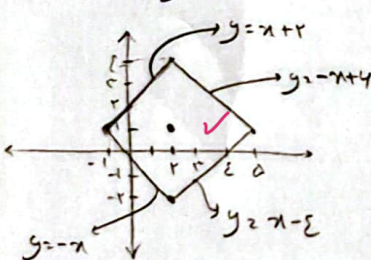
$ae - bd = 12 - 1 > 0 \rightarrow \lim_{n \rightarrow -2} y = \frac{1}{-1} = -1$



$\Rightarrow y = \left| \frac{n+1}{n+2} \right|$



$|x-2| + |y-1| = 2 \rightarrow |y-1| = 2 - |x-2|$



$|y-1| = x+1 \Rightarrow \begin{cases} y-1 = x+1 \rightarrow y = x+2 \\ y-1 = -x-1 \rightarrow y = -x \end{cases}$

$|y-1| = -x+3 \Rightarrow \begin{cases} y-1 = -x+3 \rightarrow y = -x+4 \\ y-1 = x-3 \rightarrow y = x-2 \end{cases}$

$|y-1| = 2-x \Rightarrow \begin{cases} y-1 = 2-x \rightarrow y = -x+3 \\ y-1 = x-3 \rightarrow y = x-2 \end{cases}$

الف) $f(n) = \left(\sqrt{n} + \frac{1}{n}\right)^2 + r\sqrt{n} + \frac{1}{n}$

$\min = \sqrt{r} \rightarrow r + 2\sqrt{r}$
 $\max = +\infty$
 $\frac{-b}{2a} = \frac{-r}{2} = -\frac{r}{2}$
 $\sqrt{n} \neq -1$

$\sqrt{n} + \frac{1}{n} \rightarrow (r, +\infty)$

ب) $y = (r - \sin n)(1 + \sin n) = -1(\sin n - r)(\sin n + 1)$

$y = -(\sin^2 n - r \sin n - r) \rightarrow y = -\sin^2 n + r \sin n + r$
 $\min = -1 \rightarrow -(1) + r(-1) + r = 0$
 $\max = 1 \rightarrow -1 + r(1) + r = r$
 $\frac{-b}{2a} = \frac{-r}{-2} = \frac{r}{2} \rightarrow -1 + r + \frac{r}{2}$

$R_f: [0, r]$

$\sqrt{n} + \frac{1}{n} \rightarrow (r, +\infty)$

$$f(n) = -\frac{1}{r}n + 1 \quad D_f: \{1, 2, \dots, 9\} \quad M = f(1) + f(2) + \dots + f(9)$$

$$\Rightarrow M = -\frac{1}{r}(1) + 1 - \frac{1}{r}(2) + 1 + \dots + -\frac{1}{r}(9) + 1$$

$$M = -\frac{1}{r}(1+2+\dots+9) + 9(1) = -\frac{1}{r} \left(\frac{9 \times 10}{2} \right) + 9 = \boxed{VQ}$$

$$f(n) = \sqrt{an^r + bn + c} \quad D_f: [r, +\infty) \quad \Rightarrow \begin{cases} a=0 \\ b>0 \end{cases} \quad f(n) = \sqrt{bn+c}$$

$$\Rightarrow f(r)=1 \Rightarrow f(r) = \sqrt{rb+c} = 1 \rightarrow \begin{cases} rb+c=1 \\ -rb-c=0 \end{cases} \rightarrow \begin{cases} b=1 \\ c=-r \end{cases}$$

$$g(n) = \sqrt{bn^r + an - rc} \Rightarrow g(n) = \sqrt{n^r + \varepsilon} \quad \begin{cases} \frac{b}{a} = 0 \\ y = \varepsilon \end{cases} \rightarrow R_f: \sqrt{[r, +\infty)}$$

$$f(n) = \frac{n^r - \varepsilon n}{n+r \neq 0} \rightarrow D_f: \mathbb{R} - \{a, b\} \quad R_f: [c, +\infty) - \{1\} \quad f\left(\frac{a+b}{rc}\right) = f\left(\frac{-r+\varepsilon}{-r}\right) = f(-1)$$

$$f(n) = \frac{n(n-r)(n+1)}{n+r} = n(n-r) = n^r - \varepsilon n$$

$$\begin{cases} \frac{n}{r} = \frac{-b}{10a} = \frac{r}{r} = 1 \\ y_{min} = 1 - r = -1 \end{cases} \rightarrow R_f: [-1, +\infty) - \{1\} \Rightarrow \begin{cases} c=1 \\ n \neq \varepsilon = b \Rightarrow b \geq \varepsilon \\ n \neq -r = a \end{cases}$$

$$f(n) \neq 1 \rightarrow n^r - \varepsilon n \neq 1 \rightarrow n^r - \varepsilon n - 1 \neq 0 \rightarrow (n-\varepsilon)(n+r) \neq 0$$

$$y = \sqrt{n^r + an^r + bn - 1} \quad D_f: [a, b] \quad R_f: ?$$

$$y = \sqrt{(n-r)^r} = \sqrt{n^r - 4n^r + 12n - 1} \Rightarrow \begin{cases} a=-4 \\ b=12 \end{cases}$$

$$y = n - r \rightarrow \begin{cases} n = a = -4 \rightarrow y_1 = -4 - r = -1 \\ n = b = 12 \rightarrow y_2 = 12 - r = 1 \end{cases} \quad R_f: [-1, 1]$$

$$y = [n] + [n + \frac{1}{r}] \quad D_f: [\frac{1}{r}, r)$$

$$\frac{1}{r} < n < 1 \rightarrow [\frac{1}{r}] + [\frac{1}{r} + \frac{1}{r}] = 0 + 1 = 1$$

$$1 \leq n < \frac{r}{r} \rightarrow [1] + [1 + \frac{1}{r}] = 1 + 1 = 2$$

$$\frac{r}{r} \leq n < r \rightarrow [\frac{r}{r}] + [\frac{r}{r} + \frac{1}{r}] = 1 + r = r$$

