

Examples

Arman

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1) $f(x) = \sqrt{\sin^2(x + \frac{\pi}{4}) - 1}$ $\sin^2 = 0 \rightarrow y = -1$

$\sin^2 = 1 \rightarrow y = 1$ $R = [-1, 1]$
 $\sin^2 = \frac{1}{4} \rightarrow y = \pm \frac{1}{2}$

2) $g(x) = \sqrt{\cos^2 x - 2}$ $\cos^2 = 1 \rightarrow y = 1 - 2 = -1$

$\cos^2 = 0 \rightarrow y = -2$ $R_f = [-2, -1]$
 $\cos^2 = \frac{1}{4} \rightarrow y = -2 + \frac{1}{2} = -\frac{3}{2}$

$x \neq 0 \rightarrow y = \frac{x}{|x|} + [x]$

$x < -1 \rightarrow y = -1 + x = x - 1$

$-1 < x < 1 \rightarrow y = -1 - x = -x - 1$

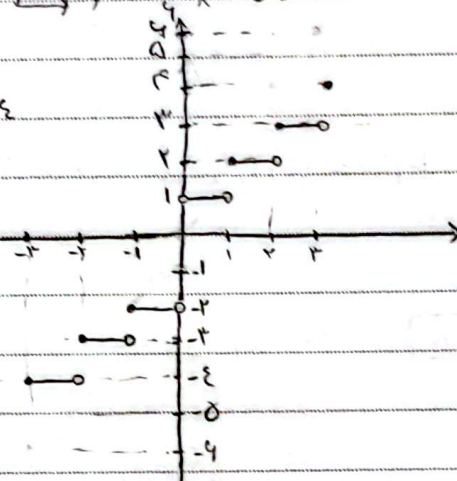
$1 < x < 2 \rightarrow y = -1 - x = -x - 1$

$x < -1 \rightarrow y = 1$

$-1 < x < 1 \rightarrow y = 1 + x = x + 1$

$1 < x < 2 \rightarrow y = 1 + x = x + 1$

$x = 2 \rightarrow y = 2$



$y = \left| \frac{x+1}{x+2} \right|$

$y = \frac{x+1}{x+2}$

$y = \left| \frac{x+1}{x+2} \right|$

$|x-1| + |y-1| = 2 \rightarrow -x+y-1=2$

$y = x+3$

$-x+y-1=2$

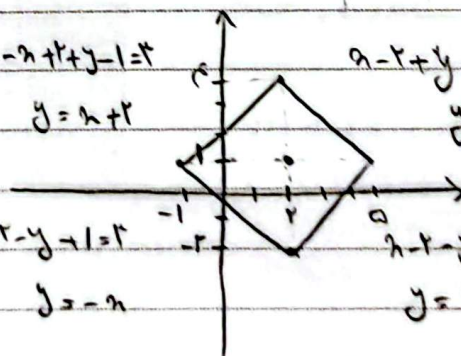
$y = x+3$

$x-y-1=2$

$y = x-3$

$x-y+1=2$

$y = x-1$



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iii) $f(x) = x + \frac{1}{x} + \sqrt{x + \frac{1}{x}}$
 $= \left(\sqrt{x + \frac{1}{x}} + 1 \right)^2 - 1$
 $x + \frac{1}{x} \geq 2 \Rightarrow y = 2 + \sqrt{2} \Rightarrow R_f = [2 + \sqrt{2}, +\infty)$
 $= [2(1 + \frac{\sqrt{2}}{2}), +\infty)$

iv) $y = (x - \sin x)(1 + \sin x) = -\sin^2 x + x \sin x + x$
 $\sin x = -1 \Rightarrow y = -1 - x + x = 0$
 $\sin x = 1 \Rightarrow y = -1 + x + x = 2x - 1$
 $\sin x = \frac{-x}{x} = -1 \Rightarrow y = -1 + x + x = 2x - 1$
 $R_f = [0, \infty)$

v) $f(x) = \frac{1}{x^2} + 1$ $D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
 $\lim_{x \rightarrow 0} f(x) = \frac{1}{x^2} (1 + x + \dots + x^9) + 9x \cdot 1 = \frac{-x^8}{x^2} + 9 = -x^6 + 9 = -18 + 9 = -9$

vi) $f(x) = \sqrt{ax^2 + bx + c}$ $[x, +\infty)$ $f(x) = \sqrt{bx + c} = 0 \Rightarrow bx + c = 0$
 $f(x) = 1 \Rightarrow \sqrt{bx + c} = 1 \Rightarrow bx + c = 1$
 $R_f = [x, +\infty)$ $b = 1 \Rightarrow c = -x$

vii) $g(x) = \sqrt{x^2 + 2}$ $\min \left| \frac{b}{2a} = 0 \right|$
 $0 + 2 = 2$
 $R_g = \sqrt{[x, +\infty)} = [x, +\infty)$

viii) $f(x) = \frac{x^2 - 4x}{x + 2} = \frac{x(x - 4)}{x + 2} = \frac{x(x + 2)(x - 4)}{x + 2} = x(x - 4)$
 $\lim_{x \rightarrow -2} f(x) = \frac{-b}{a} = \frac{4}{1} = 4$
 $1 - 4 = -3$

$R_f = [x, +\infty) - \{x\}$
 $D_f = \mathbb{R} - \{a, b\} \rightarrow a, b = -2, 2$
 $R_f = [-1, +\infty) - \{1\}$
 $\Rightarrow c = -1$

ix) $\frac{x^2 - 4x}{x + 2} = x - 2 - \frac{4}{x + 2}$
 $x^2 - 12x + 14 = 0 \Rightarrow x(x - 12) = -14 \Rightarrow x = 2$
 $f\left(\frac{a+b}{c}\right) = f\left(\frac{-1+2}{2 \times 1}\right) = f\left(\frac{1}{2}\right) = 1 - \left(\frac{1}{2}\right) = \frac{1}{2}$

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$$y = \sqrt{2x^2 + 4x + 1}$$

$$y = \sqrt{(2x+1)^2}$$

$$y = \sqrt{2x^2 + 4x + 1}$$

$$\Rightarrow a = -4, b = 1$$

Def $[-4, 1]$ $Rf = ?$

$x = -4 \rightarrow y = -1$ $Rf = [-1, 1]$

$x = 1 \rightarrow y = 1$

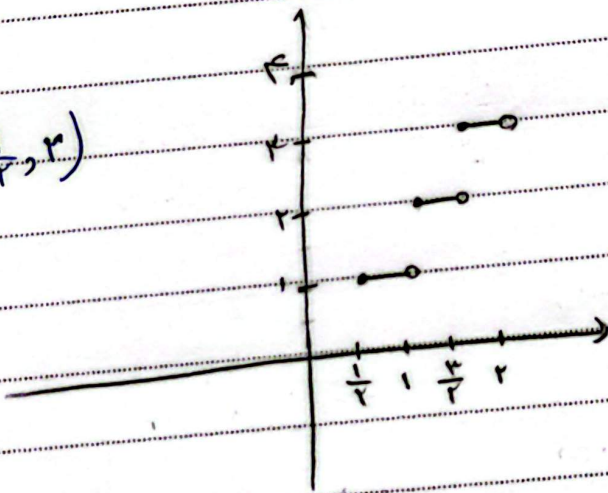
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$$y = [x] + [x + \frac{1}{x}]$$

$$\frac{1}{x} < x < 1 \rightarrow y = 0 + 1 = 1$$

$$1 < x < \frac{x}{x} \rightarrow y = 1 + 1 = 2$$

$$\frac{x}{x} < x < x \rightarrow y = 1 + x = x$$



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