

1- الف) $f(x) = 2 \sin^2(x + \frac{\pi}{4}) - 1$ $\because \sin^2(x + \frac{\pi}{4}) \leq 1$

$\rightarrow 0 \leq 2 \sin^2(x + \frac{\pi}{4}) \leq 2 \rightarrow -1 \leq f(x) \leq 1 \rightarrow R_f = [-1, 1]$

ب) $g(x) = 3 \cos^3 x - 4$

$2-1 \leq \cos^3 x \leq 1 \rightarrow -1 \leq 3 \cos^3 x - 4 \leq -1 \rightarrow R_g = [-1, -1]$

2- $y = \frac{x}{|x|} + [x]$

$[-3, -2) \rightarrow -1 - 3 = -4$

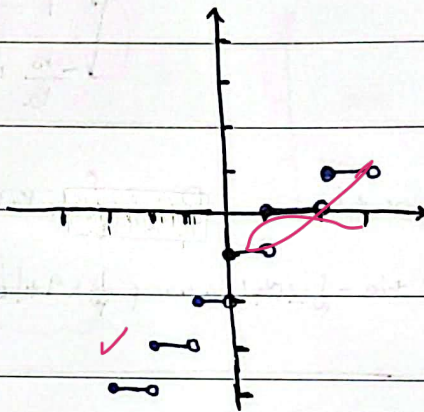
$[-2, -1) \rightarrow -1 - 2 = -3$

$[-1, 0) \rightarrow -1 - 1 = -2$

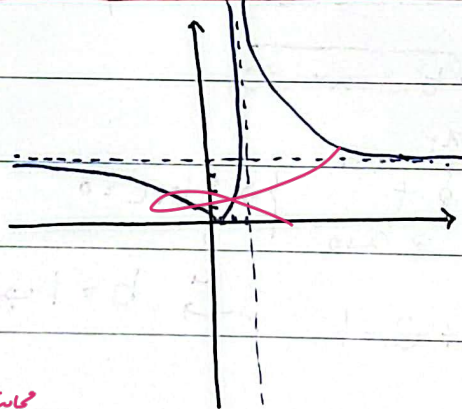
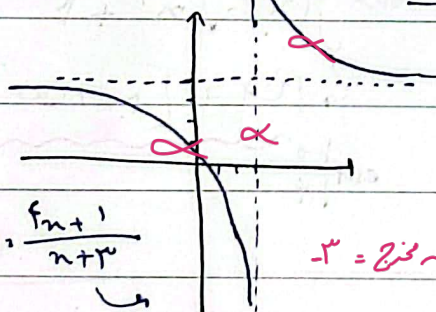
$[0, 1) \rightarrow +1 + 0 = +1$

$[1, 2) \rightarrow +1 + 1 = 2$

$[2, 3) \rightarrow +1 + 2 = 3$



3- $y = \left| \frac{x+1}{x+3} \right|$ $\frac{x+1}{x+3}$ $\frac{x+1}{x+3}$



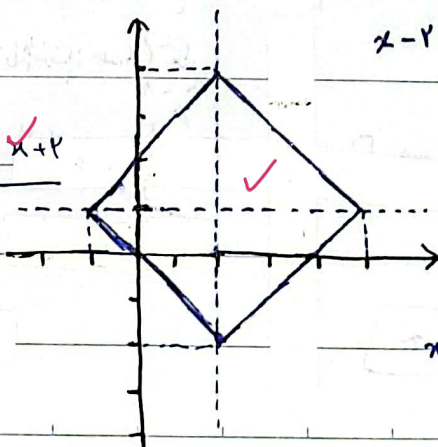
$f(x) = \frac{x+1}{x+3}$

محافظ قائم: ریشه منجمد = ۳

4- $|x-2| + |y-1| = 3$

$-x+2+y-1=3 \rightarrow y=x+2$

$-x+2-y+1=3 \rightarrow y=-x$



$x-2+y-1=3 \rightarrow y=-x+6$

$x-2-y+1=3 \rightarrow y=x-4$

2- الف) $f(x) = x + \frac{1}{x} + r\sqrt{x + \frac{1}{x}}$

$f(x) = \sqrt{x + \frac{1}{x}} + r\sqrt{x + \frac{1}{x}}$
 $0 \leq x + \frac{1}{x} \rightarrow r \leq x + \frac{1}{x}$
 $\rightarrow R_f = [r + r\sqrt{r}, +\infty)$

$\rightarrow y = (r - \sin x)(1 + \sin x)$

$\hookrightarrow y = r \sin^2 x + r \sin x + r$
 $\rightarrow R_f = [0, r]$

9- $f(x) = -\frac{1}{x}x + 10$ $D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$-\frac{1}{x}x + 10 \rightarrow -\frac{1}{x}x + 10 = -\frac{1}{x}x + 10 + \dots = -\frac{1}{x}x + 10 = -\frac{1}{x}(9 \times 10) + 90 = 90 - 10 = 80$

V- $f(x) = \sqrt{ax^2 + bx + c}$ $D_f = [r, +\infty)$ $f(r) = 1$

$g(x) = \sqrt{bx^2 + ax - rc}$

(I) $fa + rb + c = 0$

(II) $rb + c = 1$

(III) $rb + c = 1$

$\Rightarrow rb + c = 0$

$b = 1 \Rightarrow c = 1 - r$

$\Rightarrow g(x) = \sqrt{x^2 + r}$

$\hookrightarrow R_g = [r, +\infty)$

ent $\frac{0}{r}$

1- $f(x) = \frac{x^2 - f_n}{x + r}$

$D_f = \mathbb{R} \setminus \{a, b\}$

$R_f = [c, +\infty) \setminus \{1\}$

$f\left(\frac{a+b}{rc}\right)$

$f(x) = \frac{x(n+r)(n-r)}{n+r}$

$\rightarrow D_f = \mathbb{R} \setminus \{-r\}$

$\frac{n+r}{x} f(x) = x^2 - r^2 \rightarrow R = [-1, +\infty)$

$\rightarrow R_f = [-1, +\infty) \setminus \{1\}$

$2 + \frac{p(a+b)}{rc} = f(r) = 0$

$$9. y = \sqrt[n]{\underbrace{x^n}_{(n)^n} + \underbrace{ax^r}_{(-r)^n} + \underbrace{bx-1}_{(-1)^n}}$$

$$D_y = [a, b]$$

(2)

diag. $\Rightarrow y = \sqrt[n]{(n-r)^n} \Rightarrow x^n + ax^r + bx - 1 = x^n - 4x^r + 12x - 1$

$\rightarrow a = -4 \quad b = 12$

$\Rightarrow R_f = [-1, 10]$

$D_y = [-4, 12]$

$$10. y = [x] + \left[x + \frac{1}{r}\right]$$

$$\left[\frac{1}{r}, r\right)$$

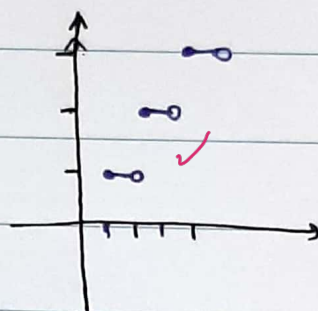
(2)

$\left[\frac{1}{r}, 1\right) \rightarrow 0 + 1 = 1$

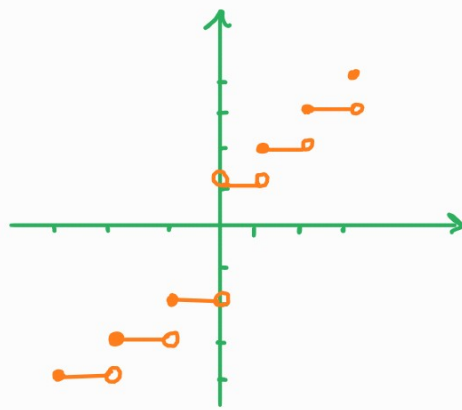
$\left[1, 1 + \frac{1}{r}\right) \rightarrow 1 + 1 = 2$

$\left[1 + \frac{1}{r}, r\right) \rightarrow 1 + r = r$

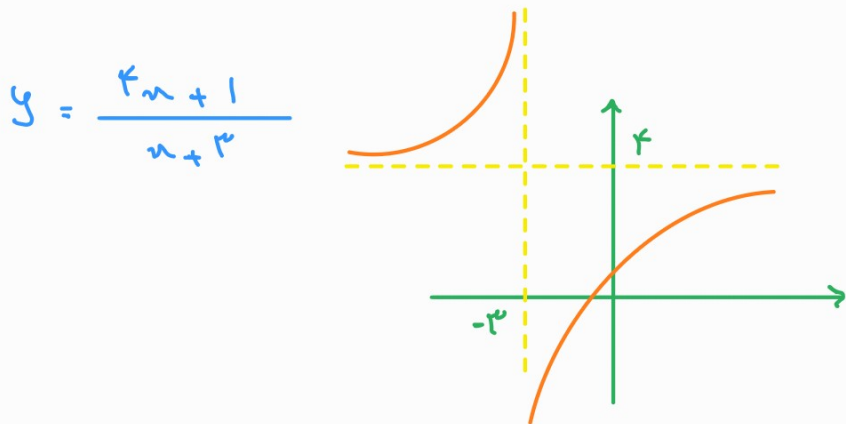
discontinuity



$$y = \begin{cases} [x] + 1 & x > 0 \\ [x] - 1 & x < 0 \end{cases} \Rightarrow$$

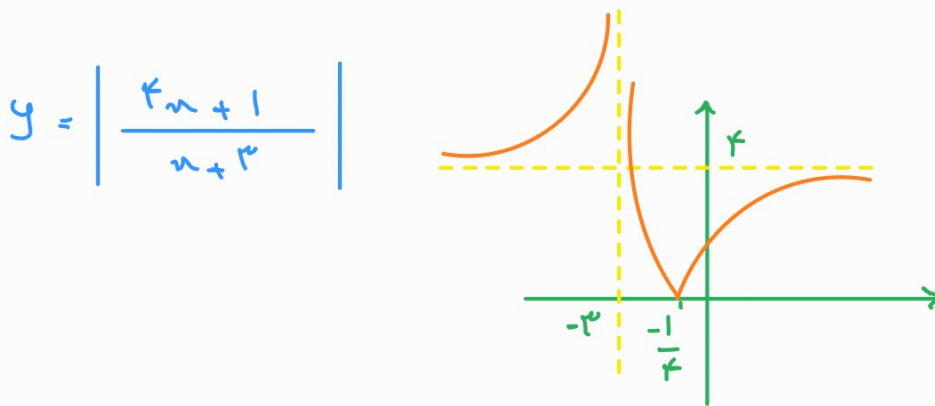


-۲



-۳

قدر مطلق دور خط، مقعر ها رو پاد میانه قدرینه
میانه بالات



$$f(x) = \frac{x(x-2)(x+2)}{x+2} = x^2 - 2x$$

-۸

$$x^2 - 2x \neq 8 \rightarrow \begin{cases} x \neq -2 \\ x \neq 4 \end{cases}$$

$y = 8$ در برد تابع نیست یعنی:

$$D_f = \mathbb{R} - \{-2, 4\}$$

پس دافند بلایر است با:

$$f(x) = x^2 - 2x \rightarrow \text{ext} \Big|_{-1}^1 \Rightarrow R_f = [-1, +\infty) - \{8\}$$

حالا بییم سراغ برد نا

$$f\left(\frac{a+b}{2}\right) = f\left(\frac{-1}{2}\right) = f(-1) \rightarrow f(-1) = 3$$