

1- ا) $f(x) = 2 \sin^2(x + \frac{\pi}{4}) - 1$ $\because \sin^2(x + \frac{\pi}{4}) \leq 1$

$\rightarrow 0 \leq 2 \sin^2(x + \frac{\pi}{4}) \leq 2 \rightarrow -1 \leq f(x) \leq 1 \rightarrow R_f = [-1, 1]$

ب) $g(x) = 3 \cos^3 x - 4$

2- $-1 \leq \cos^3 x \leq 1 \rightarrow -4 \leq 3 \cos^3 x - 4 \leq -1 \rightarrow R_g = [-4, -1]$

2- $y = \frac{x}{[x]} + [x]$

$[-3, -2) \rightarrow -1 - 3 = -4$

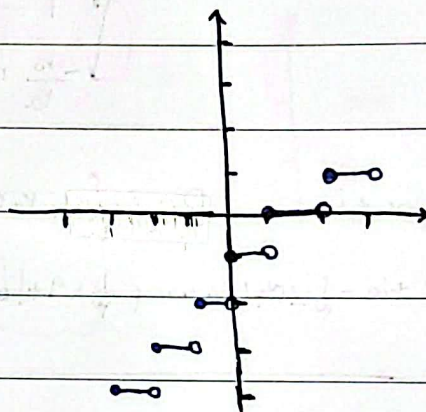
$[-2, -1) \rightarrow -1 - 2 = -3$

$[-1, 0) \rightarrow -1 - 1 = -2$

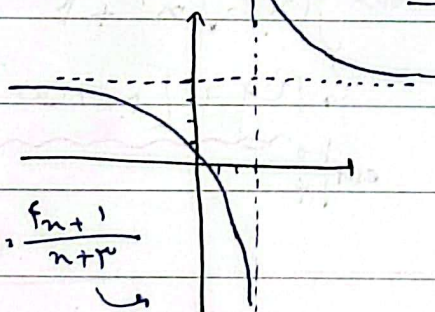
$[0, 1) \rightarrow -1 + 0 = -1$

$[1, 2) \rightarrow -1 + 1 = 0$

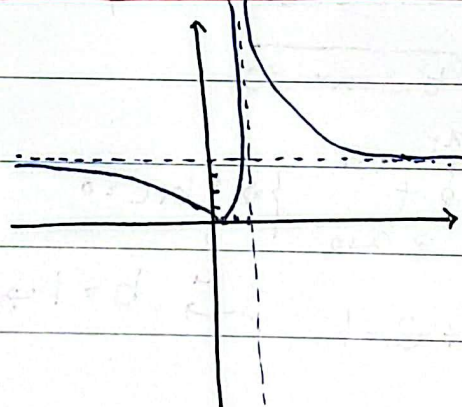
$[2, 3) \rightarrow -1 + 2 = 1$



3- $y = \left| \frac{x+1}{x+3} \right|$



180°

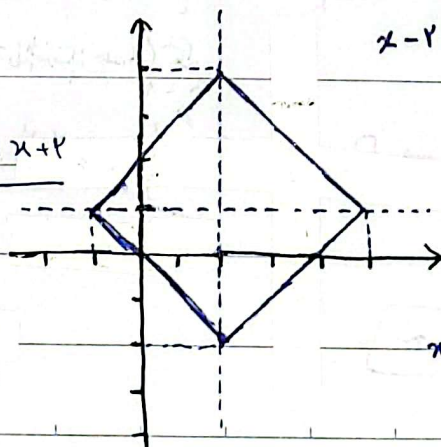


$f(x), \frac{f(x+1)}{x+3}$

4- $|x-2| + |y-1| = 3$

$-x+2+y-1=3 \rightarrow y=x+2$

$-x+2-y+1=3 \rightarrow y=-x$



$x-2+y-1=3 \rightarrow y=-x+4$

$x-2-y+1=3 \rightarrow y=x-4$

2- i) $f(x) = x + \frac{1}{x} + r\sqrt{x + \frac{1}{x}}$

$f(x) = \sqrt{x + \frac{1}{x}} + r\sqrt{x + \frac{1}{x}}$
 $0 \leq x + \frac{1}{x} \rightarrow r \leq x + \frac{1}{x} \rightarrow \left. \begin{array}{l} \sqrt{r} \rightarrow r + r\sqrt{r} \\ -\frac{b}{2a} = -1 \end{array} \right\} \Rightarrow R_f = [r + r\sqrt{r}, +\infty)$

$\rightarrow y = (r - \sin x)(1 + \sin x)$

$\hookrightarrow y = r \sin^2 x + r \sin x + r$
 $\left. \begin{array}{l} -1 \rightarrow 0 \\ 1 \rightarrow r \\ -\frac{b}{2a} = 1 \end{array} \right\} \Rightarrow R_f = [0, r]$

9- $f(x) = -\frac{1}{p}x + 10$ $D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$-\frac{1}{p} \times 1 + 10 \rightarrow -\frac{1}{p} \times 9 + 10 \rightarrow -\frac{1}{p} \times 7 + 10 \rightarrow \dots \rightarrow -\frac{1}{p} \times 1 + 10 = -\frac{1}{p} \left(\frac{9 \times 10}{p} \right) + 90 = 90 - 10 = 80$

V- $f(x) = \sqrt{ax^2 + bx + c}$ $D_f = [r, +\infty)$ $f(r) = 1$

$g(x) = \sqrt{bx^2 + ax - rC}$

(I) $fa + rb + c = 0$

(II) $rb + c = 0$

(III) $rb + c = 1$

$\Rightarrow g(x) = \sqrt{x^2 + r}$

$\hookrightarrow R_g = [r, +\infty)$

ent $\left| \begin{array}{l} 0 \\ r \end{array} \right.$

1- $f(x) = \frac{x^2 - tx}{x + r}$

$D_f = \mathbb{R} \setminus \{a, b\}$

$R_f = [c, +\infty) \setminus \{1\}$

$f\left(\frac{a+b}{rC}\right)$

$f(x) = \frac{x(n+r)(n-r)}{n+r}$

$\rightarrow D_f = \mathbb{R} \setminus \{-r\}$

$\frac{n+r}{x} f(x) = x^2 - rn \rightarrow R = [-1, +\infty)$

$\rightarrow R_f = [-1, +\infty) \setminus \{1\}$

$2 + \frac{p(a+b)}{rC} = f(r) = 0$

9. $y = \sqrt[n]{\underbrace{x^n + ax^r + bx - 1}_{(n)^n} \underbrace{}_{(-r)^n}}$ $D_y = [a, b]$

diag. $\Rightarrow y = \sqrt[n]{(n-r)^n} \Rightarrow x^n + ax^r + bx - 1 = x^n - 4x^r + 12x - 1$

$\rightarrow a = -4 \quad b = 12 \Rightarrow R_f = [-1, 10]$
 $D_y = [-4, 12]$

10. $y = [x] + [x + \frac{1}{r}] \quad [\frac{1}{r}, 2)$

$[\frac{1}{r}, 1) \rightarrow 0 + 1 = 1$

$[1, 1 + \frac{1}{r}) \rightarrow 1 + 1 = 2$

$[1 + \frac{1}{r}, 2) \rightarrow 1 + 2 = 3$

discontinuity

