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تاریخ امتحان تکلیف شماره ۶

نمونه دخت ۱

$$f(x) = 2 \sin^2\left(x + \frac{\pi}{3}\right) - 1$$

$$-1 \leq \sin\left(x + \frac{\pi}{3}\right) \leq 1 \xrightarrow{\text{توان ۲}} 0 \leq \sin^2\left(x + \frac{\pi}{3}\right) \leq 1$$

$$\xrightarrow{\text{ضرب ۲ و منهای ۱}} -1 \leq 2 \sin^2\left(x + \frac{\pi}{3}\right) - 1 \leq 1$$

$$R_f = [-1, 1]$$

$$g(x) = 3 \cos^3 x - 4$$

$$-1 \leq \cos x \leq 1 \xrightarrow{\text{توان ۳}} -1 \leq \cos^3 x \leq 1 \xrightarrow{\text{ضرب ۳ و منهای ۴}} -7 \leq 3 \cos^3 x - 4 \leq -1$$

$$R_g = [-7, -1]$$

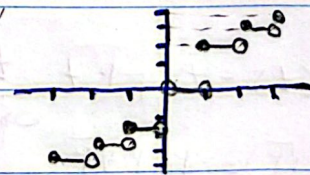
$$y = \frac{x}{|x|} + [x]$$

 $x \neq 0$

$$x < 0 \rightarrow y = -1 + [x] \xrightarrow{\begin{matrix} [x] = -3 \rightarrow [-3, -2) \rightarrow -4 = -4 \\ [x] = -2 \rightarrow [-2, -1) \rightarrow -3 = -3 \\ [x] = -1 \rightarrow [-1, 0) \rightarrow -2 = -2 \end{matrix}}$$

$$x > 0 \rightarrow y = 1 + [x] \xrightarrow{\begin{matrix} [x] = 0 \rightarrow (0, 1) \rightarrow 1 = 1 \\ [x] = 1 \rightarrow [1, 2) \rightarrow 2 = 2 \\ [x] = 2 \rightarrow [2, 3) \rightarrow 3 = 3 \end{matrix}}$$

$$\text{IF } x=0 \rightarrow y = 1 + 0 = 1 \rightarrow [x] = 0$$



Date: ^{تاريخ} $x = -\frac{1}{k}$ Sub:

$$y = \left| \frac{kx+1}{x+k} \right| \rightarrow \frac{k(x+k)-11}{x+k} \rightarrow k - \frac{11}{x+k} \rightarrow x = -k \rightarrow y = k$$

برضه! $\rightarrow \frac{1}{k} = y$

$x = -k \rightarrow y = k$

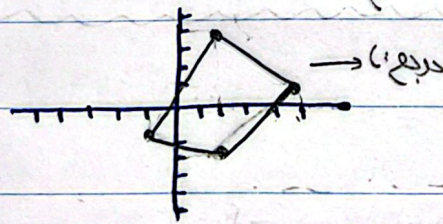
نقطه صاف افق:

$$|x-2| + |y-1| = 3 \rightarrow |y-1| = 3 - |x-2| \geq 0 \rightarrow |x-2| \leq 3 \rightarrow -1 \leq x \leq 5$$

$$\rightarrow y-1 = \pm (3 - |x-2|) \rightarrow y = 1 \pm (3 - |x-2|)$$

$$y = 3 - |x-2| \rightarrow y = \begin{cases} x+1 & -1 \leq x \leq 2 \\ -x+5 & 2 < x \leq 5 \end{cases}$$

$$y = -2 + |x-2| \rightarrow y = \begin{cases} -x & -1 \leq x \leq 2 \\ x-4 & 2 < x \leq 5 \end{cases}$$



الف) $F(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$ $x > 0 \rightarrow x + \frac{1}{x} \geq 2$

$\rightarrow x + \frac{1}{x} \geq 0 \rightarrow x > 0$

$\rightarrow \sqrt{x + \frac{1}{x}} = t \rightarrow t \geq \sqrt{2}$

$F(t) = t^2 + 2t, t \geq \sqrt{2}$

$F(\sqrt{2}) = 2 + 2\sqrt{2}$

$\Rightarrow R_F = [2 + 2\sqrt{2}, +\infty)$

$$b) y = (1 - \sin x)(1 + \sin x) \xrightarrow{\sin x = t} t \in [-1, 1]$$

$$y = (1 - t)(1 + t) = 1 + t - t^2 = 1 - (t - 1)^2$$

$$t = 1 \rightarrow y_{\max} = 1 - 0 = 1$$

$$t = -1 \rightarrow y_{\min} = 1 - (-2)^2 = -3$$

$$R_y = [-3, 1]$$

$$f(x) = -\frac{1}{\pi}x + 10 \rightarrow D_f = \{1, 2, \dots, 97\}$$

$$1 \times 9 = 9 \rightarrow 9 - \frac{1}{\pi}(-f(x)) = 9 - 10 = -1$$

$$f(x) = \sqrt{ax^2 + bx + c} \rightarrow [r, +\infty) \xrightarrow{a=0} \sqrt{bx+c}$$

$$f(r) = 1$$

$$g(x) = \sqrt{bx^2 + ax - rc}$$

$$\sqrt{bx+c} [r, +\infty) \rightarrow b > 0, -\frac{c}{b} = r \rightarrow c = -rb \rightarrow f(x) = \sqrt{b(x-r)}$$

$$\rightarrow f(r) = 1 \rightarrow \sqrt{b(r-r)} = \sqrt{b} = 1 \rightarrow b = 1, c = -r$$

$$\rightarrow g(x) = \sqrt{x^2 + r} \rightarrow x^2 + r \geq r \rightarrow \sqrt{x^2 + r} \geq r$$

$$\rightarrow R_g = [r, +\infty)$$

$$f(x) = \frac{x^2 - 4x}{x+2} \rightarrow D_f = \mathbb{R} - \{a, b\} \quad \underline{1}$$

$$\rightarrow R_f = [c, +\infty) - \{n\}$$

$$f\left(\frac{a+b}{2c}\right) \Rightarrow \left(\frac{-2+4}{2(-1)}\right) = \frac{2}{-2} = \underline{-1}$$

$$f(x) = \frac{x(x-2)(x+2)}{x+2} = x^2 - 2x \rightarrow \text{Graph} \rightarrow x=1 \text{ only}$$

$$\rightarrow y_{\min} = -1 \rightarrow c = -1$$

$$f(-1) = (-1)^2 - 2(-1) = 1 + 2 = 3$$

$$x = -2 \rightarrow (-2)^2 - 2(-2) = 4 + 4 = 8$$

$$x^2 - 2x = 1 \rightarrow x \neq 4$$

$$R = \{-2, 4\}$$

$$y = \sqrt[3]{x^3 + ax^2 + bx - 1} \rightarrow D_y = [a, b] \quad \underline{9}$$

$$(x-2)^3 = x^3 - 6x^2 + 12x - 8 \rightarrow a = -6 \text{ and } b = 12$$

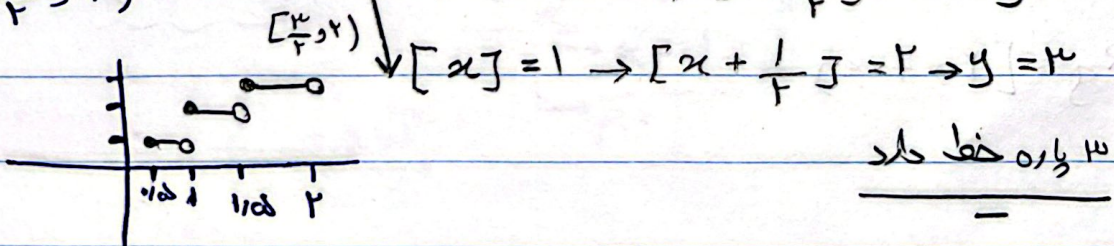
$$y = x-2 \rightarrow [a, b] = [-6, 12] \rightarrow x = -6 \rightarrow y = -6-2 = -8$$

$$x = 12 \rightarrow y = 12-2 = 10$$

$$\rightarrow \underline{[-8, 10]}$$

$$y = [x] + \left[x + \frac{1}{x}\right] \rightarrow [x] = 0 \rightarrow \left[x + \frac{1}{x}\right] = 1 \rightarrow y = 1 \quad \underline{10}$$

$$\left[\frac{1}{x}, 2\right) \rightarrow [x] = 1 \rightarrow \left[x + \frac{1}{x}\right] = 1 \rightarrow y = 2$$



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