

الف) $f(u) = 2\sin^2(u + \frac{\pi}{3}) - 1$
 $-1 \leq \sin(u + \frac{\pi}{3}) \leq 1 \Rightarrow 0 \leq \sin^2(u + \frac{\pi}{3}) \leq 1 \Rightarrow 0 \leq 2\sin^2(u + \frac{\pi}{3}) \leq 2 \Rightarrow -1 \leq 2\sin^2(u + \frac{\pi}{3}) - 1 \leq 1$
 $\Rightarrow R_f = [-1, 1] \rightarrow$ جواب

ب) $g(u) = 3\cos^3 u - 2$
 $-1 \leq \cos u \leq 1 \Rightarrow -1 \leq \cos^3 u \leq 1 \Rightarrow -3 \leq 3\cos^3 u \leq 3 \Rightarrow -5 \leq 3\cos^3 u - 2 \leq 1$
 $\Rightarrow R_g = [-5, 1] \rightarrow$ جواب

* در بازه های منفی $\frac{u}{|u|}$ می شود $-\frac{u}{|u|}$ یعنی 1- و در بازه های مثبت $\frac{u}{|u|}$ می شود $\frac{u}{|u|}$ یعنی 1
 و u نباید صفر باشد (برای $\frac{u}{|u|}$ و $-\frac{u}{|u|}$ معرّفی شده و خروج کسری $\frac{u}{|u|}$ تعریف نشده می شود)

$y = \frac{u}{|u|} + [u]$
 $-3 \leq u \leq -2 \rightarrow [u] = -3 \rightarrow y = -1 - 3 = -4$
 $-2 \leq u \leq -1 \rightarrow [u] = -2 \rightarrow y = -1 - 2 = -3$
 $-1 \leq u \leq 0 \rightarrow [u] = -1 \rightarrow y = -1 - 1 = -2$
 $0 \leq u \leq 1 \rightarrow [u] = 0 \rightarrow y = 1$
 $1 \leq u \leq 2 \rightarrow [u] = 1 \rightarrow y = 1 + 1 = 2$
 $2 \leq u \leq 3 \rightarrow [u] = 2 \rightarrow y = 1 + 2 = 3$
 $u = 3 \rightarrow [u] = 3 \rightarrow y = 1 + 3 = 4$

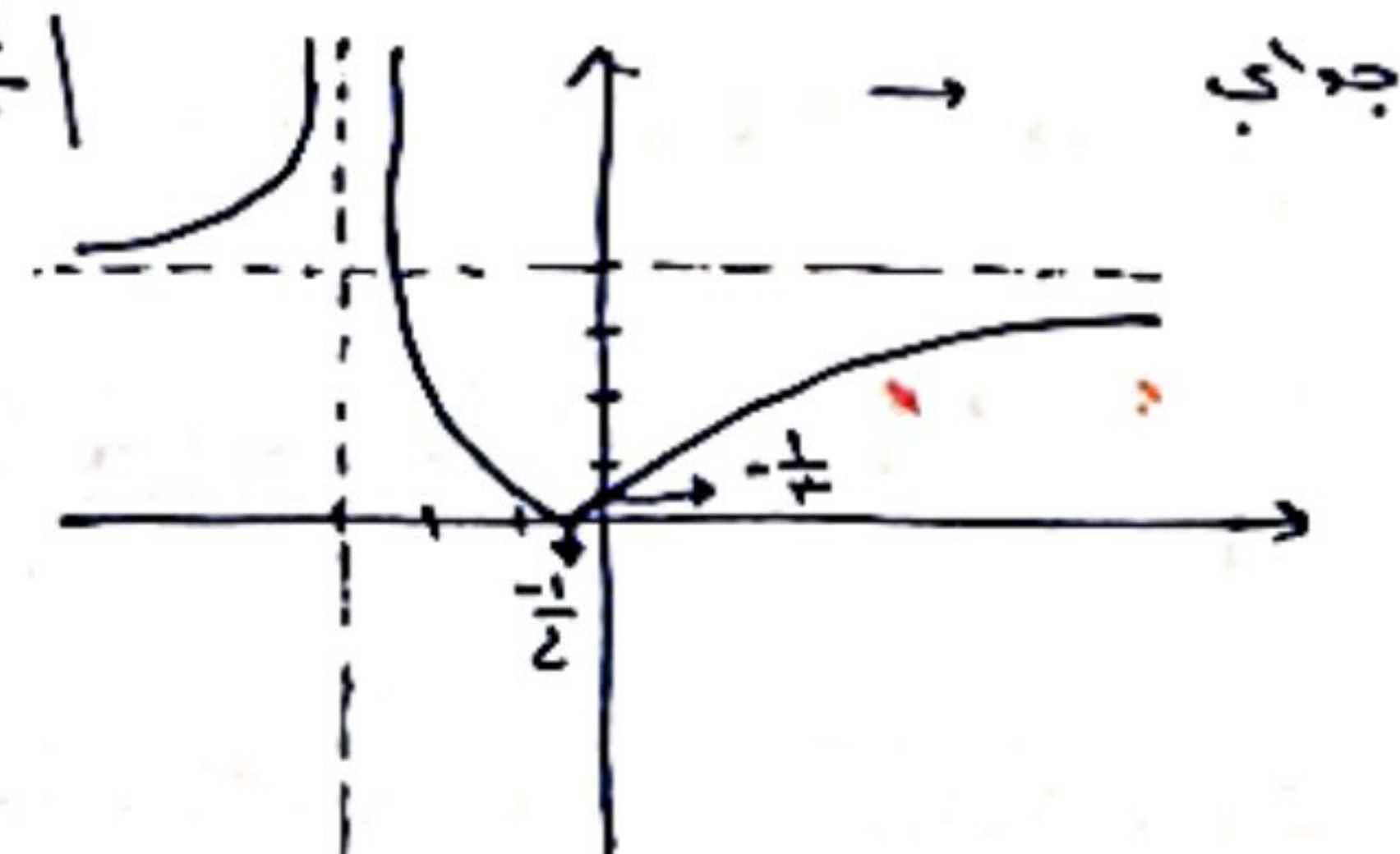
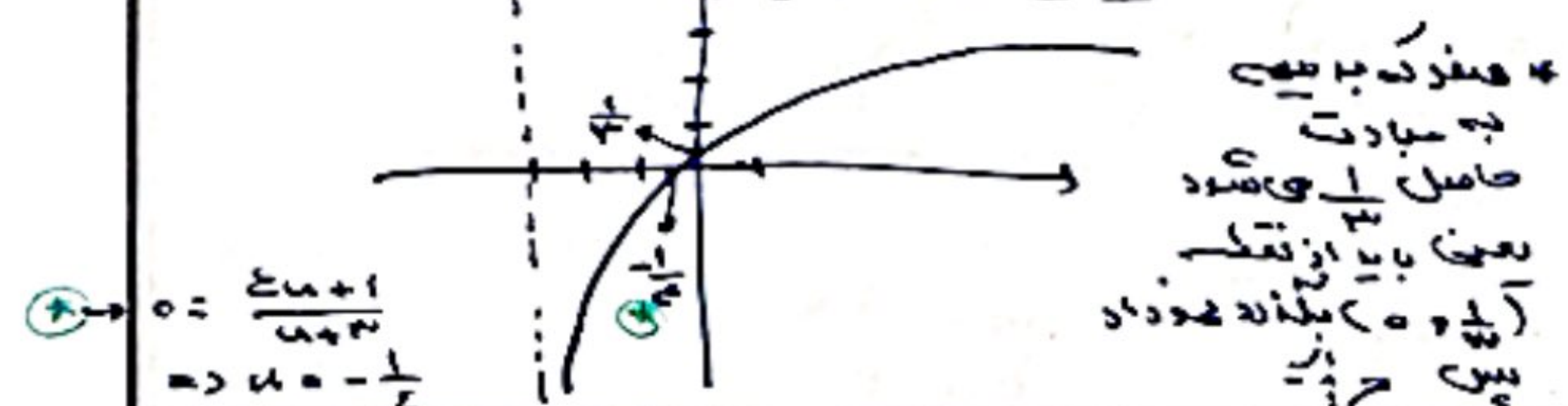


$y = \left| \frac{u+1}{u+3} \right|$

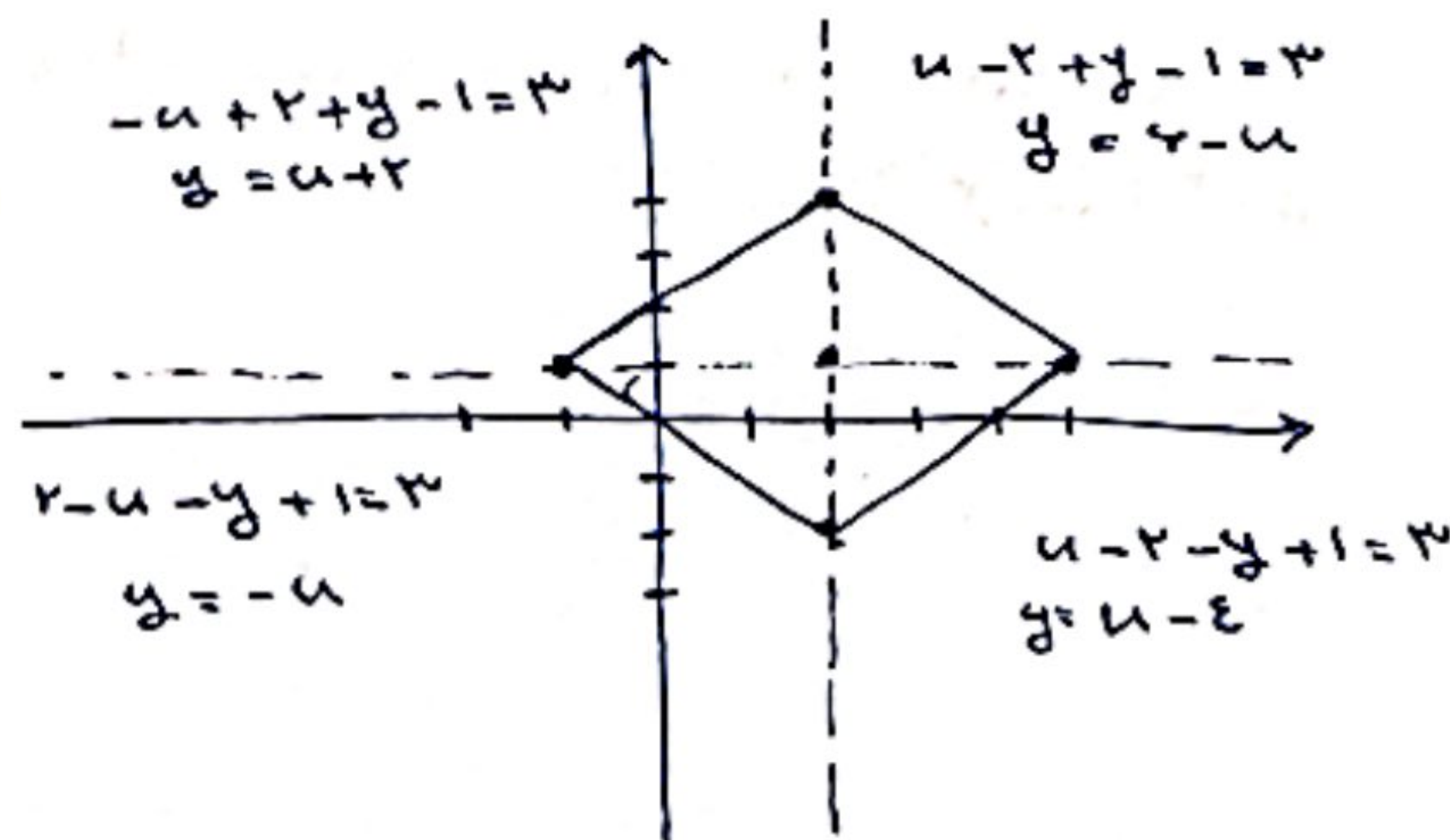
① $y = \frac{u+1}{u+3}$

در بازه های منفی $\frac{u}{|u|}$ می شود $-\frac{u}{|u|}$ یعنی 1-
 و در بازه های مثبت $\frac{u}{|u|}$ می شود $\frac{u}{|u|}$ یعنی 1

چون قدر مطلق دور یک عبارت است و همیشه از صفر بزرگتر یا مساوی است
 ② $y = \left| \frac{u+1}{u+3} \right|$
 می افتاد بالای محور ها



$|u-2| + |y-1| = 3$



الف) $f(u) = u + \frac{1}{u} + 2\sqrt{u + \frac{1}{u}} \Rightarrow f(u) = \left(\sqrt{u + \frac{1}{u}}\right)^2 + 2\sqrt{u + \frac{1}{u}} \Rightarrow \sqrt{u + \frac{1}{u}} + 2 = \frac{p}{q} = \frac{r}{s}$
 $u + \frac{1}{u} \geq 0$
 $u > 0 \Rightarrow u + \frac{1}{u} \geq 2$
 $\Rightarrow R_f = [2 + 2\sqrt{2}, +\infty) \rightarrow$ جواب

ب) $y = (3 - \sin u)(1 + \sin u) \Rightarrow y = -\sin^2 u + 2\sin u + 3 \Rightarrow \sin u = \frac{p}{q} = \frac{r}{s} = 1$
 $\Rightarrow R_g = [0, 3] \rightarrow$ جواب

$$f(u) = \frac{1}{u}u + 10 \rightarrow D_f = \{1, \dots, 9\}$$

$$-\frac{1}{u} \times 1 + 10 + -\frac{1}{u} \times 2 + 10 + \dots + -\frac{1}{u} \times 9 + 10 = -\frac{1}{u} (1+2+\dots+9) + 9 \times 10 = \frac{45 \times -1}{u} + 90$$

$$* \text{ } 2 \times 10 = \frac{9 \times 10}{u} = 20$$

(Vas)

جواب

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$$f(u) = \sqrt{au^2 + bu + c} \rightarrow D_f = [2, +\infty)$$

$$\xrightarrow{a=0} f(u) = \sqrt{bu + c} \quad \begin{cases} u=2 \rightarrow b+c=0 \\ f(2)=1 \rightarrow 2b+c=1 \end{cases} \Rightarrow \underline{b=1}, \underline{c=-2}$$

نکته: $f(u)$ ریشه معادله داشته باشد R به دست آید Δ و Δ درجه دو دارد دو ریشه داشته باشد و این ریشه ها $u \geq 2$ باشد $a=0$ به دست آید

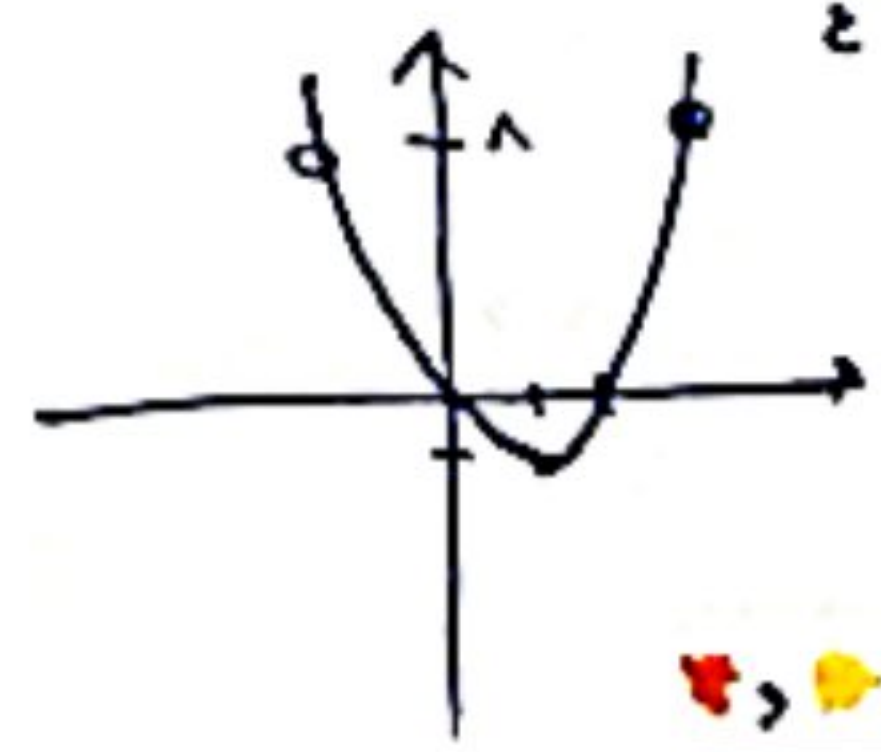
$$g(u) = \sqrt{bu^2 + au - c} \xrightarrow{a=0} g(u) = \sqrt{u^2 + 1} \rightarrow y' = u^2 + 1 \rightarrow \text{ext} \left| \begin{array}{l} -\frac{b}{2a} = u = 0 \\ y = 1 \end{array} \right. \xrightarrow{a \geq 0} R_g = [1, +\infty)$$

$$\Rightarrow R_g(u) = \sqrt{[2, +\infty)} \Rightarrow [2, +\infty) \rightarrow \text{جواب}$$

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$$f(u) = \frac{u^2 - 2u}{u+2} \Rightarrow \frac{u(u-2)}{u+2} \Rightarrow \frac{u(u-2)(u+2)}{u+2} \Rightarrow f(u) = u^2 - 2u$$

$$\xrightarrow{if u=2} f(2) = \frac{(-2)^2 - 2 \times 2}{2} = 0 \Rightarrow u^2 - 2u = 0 \Rightarrow (u+2)(u-2) = 0 \Rightarrow D_f = \mathbb{R} - \{-2, 2\}$$



$$R_f(u) \Rightarrow f(u) = u^2 - 2u \rightarrow \text{ext} \left| \begin{array}{l} u = -\frac{b}{2a} = \frac{2}{2} = 1 \\ y = 1 - 2 \times 1 = -1 \end{array} \right. \xrightarrow{a \geq 0} R_f = [-1, +\infty) - \{1, 2\}$$

$$\xrightarrow{f\left(\frac{a+b}{2}\right)} f\left(\frac{-2+2}{-2}\right) \Rightarrow f(-1) = (-1)^2 - 2 \times (-1) = 1$$

$$y = \sqrt[3]{u^3 + au^2 + bu - 1} \xrightarrow{a=0} y = \sqrt[3]{(u-2)^3} \Rightarrow y = \sqrt[3]{u^3 - 4u^2 + 12u - 1} \Rightarrow a = -4, b = 12$$

$$D_y = [a, b] \xrightarrow{a=0} D_y = [-4, 12] \rightarrow -4 \leq u \leq 12 \rightarrow -1 \leq u-2 \leq 10 \Rightarrow R_f = [-1, 10]$$

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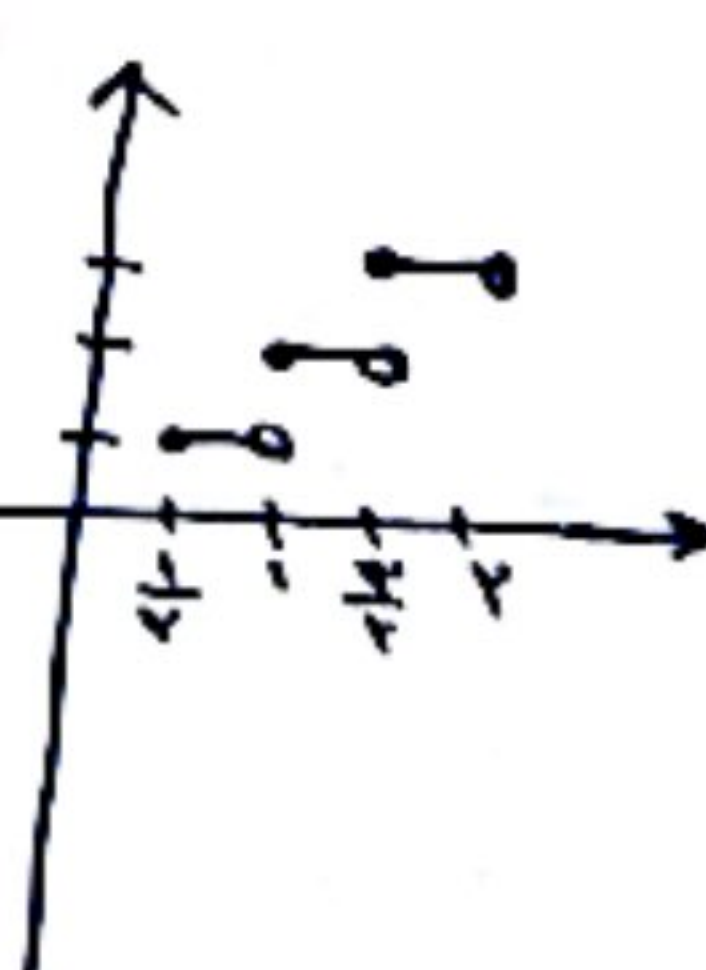
$$y = [u] + [u + \frac{1}{2}]$$

$$\frac{1}{2} \leq u < 1 \rightarrow 1 \leq u + \frac{1}{2} < \frac{3}{2} \Rightarrow [u] = 0, [u + \frac{1}{2}] = 1 \rightarrow 0 + 1 = 1$$

$$1 \leq u < \frac{3}{2} \rightarrow \frac{3}{2} \leq u + \frac{1}{2} < 2 \Rightarrow [u] = 1, [u + \frac{1}{2}] = 1 \rightarrow 1 + 1 = 2$$

$$\frac{3}{2} \leq u < 2 \rightarrow 2 \leq u + \frac{1}{2} < \frac{5}{2} \Rightarrow [u] = 1, [u + \frac{1}{2}] = 2 \rightarrow 1 + 2 = 3$$

جواب: $\{1, 2, 3\}$



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