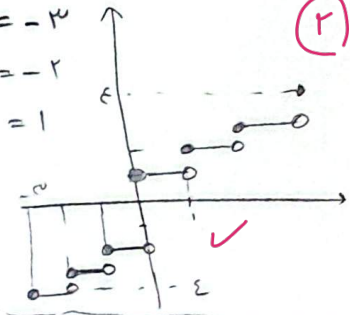


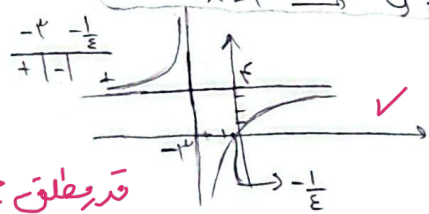
۱) الف)  $f(x) = 2\sin^2(x + \frac{\pi}{3}) - 1$   $\rightarrow 0 \leq \sin^2(x + \frac{\pi}{3}) \leq 1 \xrightarrow{\times 2} 0 \leq 2\sin^2(x + \frac{\pi}{3}) \leq 2 \xrightarrow{-1} -1 \leq 2\sin^2(x + \frac{\pi}{3}) - 1 \leq 1 \rightarrow R_f = [-1, 1]$  (۲)

ب)  $g(x) = 3\cos^3 x - 4$   
 $-1 \leq \cos^3 x \leq 1 \rightarrow -3 \leq 3\cos^3 x \leq 3 \rightarrow -3 - 4 = -7 \rightarrow 3 - 4 = -1 \rightarrow R_g = [-7, -1]$

۲) نمودار  $y = \frac{x}{|x|} + [x]$  در بازه  $[-3, 3]$   
 $-3 \leq x < -2 \rightarrow [x] = -3 \rightarrow y = -1 - \epsilon = -\epsilon \rightarrow [-3, -2) \rightarrow y = -\epsilon$   
 $-2 \leq x < -1 \rightarrow [x] = -2 \rightarrow y = -3$   
 $-1 \leq x < 0 \rightarrow [x] = -1 \rightarrow y = -2$   
 $0 \leq x < 1 \rightarrow [x] = 0 \rightarrow y = 1$   
 $1 \leq x < 2 \rightarrow y = 2$   
 $2 \leq x < 3 \rightarrow y = 3$   
 $x = 3 \rightarrow y = 4$

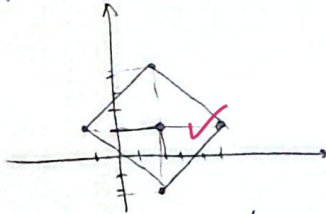


۳) نمودار  $y = \frac{x+1}{x+3}$   
 $x+3 \neq 0 \rightarrow x \neq -3$   
 $x+1 = 0 \rightarrow x = -1$



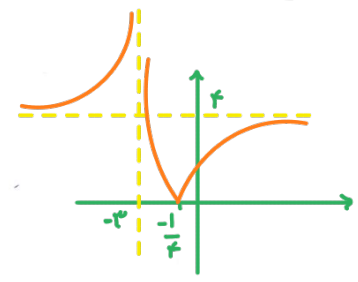
قد مطلق چه شد!

۴) نمودار  $|x-2| + |y-1| = 3$   
 منطقه مربعی



با ضابطه بندی!!

$y = \left| \frac{x+1}{x+3} \right|$



$x + \frac{1}{x} \geq 0 \rightarrow \frac{x^2 + 1}{x} \geq 0$   
 $x > 0 \rightarrow x + \frac{1}{x} \geq 2$   
 $t > 1$  (۲)

الف)  $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$   
 $t \rightarrow [2, +\infty) \xrightarrow{\min} 2 + 2\sqrt{2} \rightarrow [2 + 2\sqrt{2}, +\infty)$

ب)  $y = (3 - \sin x)(1 + \sin x) \rightarrow 3 + 3\sin x - \sin x - \sin^2 x$   
 $- \sin^2 x + 2\sin x + 3$   
 $\sin x \rightarrow [-1, 1]$

max  $-1 + 2 + 3 = 4$   
 min  $-(1) + 2(-1) + 3 = 0$   
 $-\frac{b}{2a} = -\frac{2}{-2} = 1 \rightarrow \leq$   
 $[0, 4]$

۴)  $f(x) = -\frac{1}{x} + 1 \rightarrow D_f = [1, 9] \rightarrow R_f = ?$

$f(1) = 1 - \frac{1}{1} = 0$   
 $f(2) = 1 - \frac{1}{2} = \frac{1}{2}$   
 $f(9) = 1 - \frac{1}{9} = \frac{8}{9}$   
 $\rightarrow \frac{19}{4} + \frac{17}{4} + \dots + \frac{11}{4} = \frac{295 - 11}{4} = \frac{284}{4} = 71$   
 $\frac{1}{4} (19 + 17 + \dots + 11) \Rightarrow \frac{295}{4} = 73.75$  (۲)

$$v) f(x) = \sqrt{ax^2 + bx + c} \rightarrow [r, +\infty) \xrightarrow{x=r} \varepsilon a + r b + c = 0 \rightarrow$$

$$f(r) = 1$$

$$R_f = ? \rightarrow g(x) = \sqrt{bx^2 + ax - rc}$$

$$\begin{cases} b = -\varepsilon a \\ c = \varepsilon a \end{cases}$$

$$f(r) = 1 \rightarrow 1 = \sqrt{a(r-r)^2} \rightarrow 1 = \sqrt{a} \rightarrow a = 1$$

$$\alpha = 1$$

$$\begin{cases} b = -\varepsilon \\ c = \varepsilon \end{cases} \rightarrow g(x) = \sqrt{-\varepsilon x^2 + x - 1} \rightarrow -\varepsilon x^2 + x - 1 \geq 0 \rightarrow \Delta < 0$$

$$D_g = \emptyset$$

$$n) f(x) = \frac{n(x-r)(x+r)}{n(x^2-\varepsilon)} = \frac{x^2 - \varepsilon}{x+r}$$

$$x \neq -r$$

$$D_f = \mathbb{R} - \{a, b\}$$

$$R_f = [c, +\infty) - \{1\}$$

$$f\left(\frac{a+b}{rc}\right) = \frac{r-r}{r(-1)} = 1$$

$$f(r) = \frac{r^2 - \varepsilon(r)}{r+r} = \frac{r^2 - \varepsilon(r)}{2r}$$

$$f(x) = \frac{x(x-r)(x+r)}{x^2 - \varepsilon} \xrightarrow{\text{دالة}} (x-1)^2 - 1 \text{ ! تغيير !} \rightarrow D_f = \mathbb{R} - \{-1\}$$

$$(x-1)^2 \geq 0 \rightarrow f(x) \geq -1 \xrightarrow{\min c} c = -1$$

$$f(-r) = (-r)^2 - r(-r) = 1 \rightarrow R_f = [-1, +\infty) - \{1\}$$

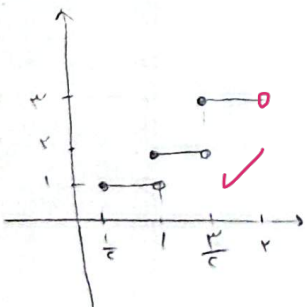
$$q) y = \sqrt{x^2 + ax^2 + bx - 1} \rightarrow D_y = [a, b] \rightarrow R_y = ? \underline{R_0^2}$$

$$D_y = \mathbb{R} \rightarrow R_y =$$

المتغير  $x$  يتغير بين  $a$  و  $b$  ، فماذا يتغير  $y$  ؟

المتغير  $x$  يتغير بين  $a$  و  $b$  ، فماذا يتغير  $y$  ؟

$$1. y = [x] + \left[x + \frac{1}{r}\right] \rightarrow D_f = \left[\frac{1}{r}, r\right) \xrightarrow{\text{دالة}} \underline{\underline{0, 1, 2}}$$



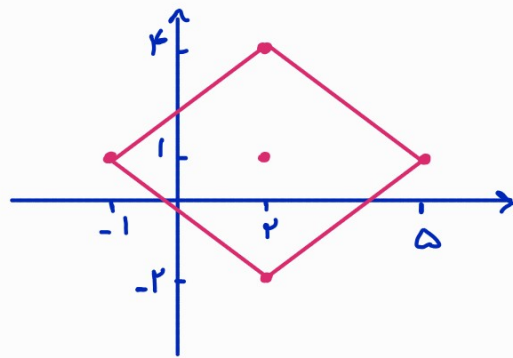
$$\frac{1}{r} \leq x < 1 \rightarrow [x] = 0$$

$$1 \leq x + \frac{1}{r} < \frac{r}{r} \rightarrow \left[x + \frac{1}{r}\right] = 1 \rightarrow y = 1$$

$$1 \leq x < \frac{r}{r} \rightarrow [x] = 1 \rightarrow \left[x + \frac{1}{r}\right] = 1 \rightarrow y = 2$$

$$\frac{r}{r} \leq x < r \rightarrow [x] = 1 \rightarrow \left[x + \frac{1}{r}\right] = 2 \rightarrow y = 3$$

$$\begin{cases} x > 2, y > 1 \rightarrow y = -x + 4 \\ x > 2, y \leq 1 \rightarrow y = x - 4 \\ x \leq 2, y > 1 \rightarrow y = x + 2 \\ x \leq 2, y \leq 1 \rightarrow y = -x \end{cases}$$



۴-

۷- با توجه به دامنه، زیر رادیکال یک خط با شیب مثبت بوده یعنی  $a = 0$  است و نقطه  $(2, 0)$  هم در معادله خط صدق می‌کند...

$$\begin{cases} (2, 0) \rightarrow 2b + c = 0 \\ (3, 1) \rightarrow 3b + c = 1 \end{cases} \rightarrow b = 1, c = -2$$

$$g(x) = \sqrt{x^2 + 4} \rightarrow \text{ext} \Big|_x^0 \Rightarrow R_g = \sqrt{[4, +\infty)} = [2, +\infty)$$

$$f(x) = \frac{x(x-2)(x+2)}{x+2} = x^2 - 2x$$

۸-

$$x^2 - 2x \neq 8 \rightarrow \begin{cases} x \neq -2 \\ x \neq 4 \end{cases}$$

$y = 8$  در برد تابع نیست یعنی:

$$D_f = \mathbb{R} - \{-2, 4\}$$

پس دامنه برابر است با:

$$f(x) = x^2 - 2x \rightarrow \text{ext} \Big|_{-1}^1 \Rightarrow R_f = [-1, +\infty) - \{8\}$$

حالا بیایم سراغ برد:

$$f\left(\frac{a+b}{2c}\right) = f\left(\frac{2}{-2}\right) = f(-1) \rightarrow f(-1) = 3$$

9. چون عدد ثابت عبارت زیر را دیکال،  $-۸$  است پس حتماً  $(x-۲)^۳$  بوده و داریم :

$$y = \sqrt[۳]{x^۳ - 4x^۲ + 12x - ۸} \rightarrow a = -4, b = 12 \rightarrow D_y = [-4, 12]$$

$$x = -4 \rightarrow y = -۸$$

$$x = 12 \rightarrow y = ۱۰$$

$$\rightarrow R_y = [-۸, ۱۰]$$