

1)

الف) $f(x) = 2\sin^2(x + \frac{\pi}{4}) - 1 \rightarrow 0 \leq \sin^2(x + \frac{\pi}{4}) \leq 1 \xrightarrow{\times 2} 0 \leq 2\sin^2(x + \frac{\pi}{4}) \leq 2 \rightarrow -1 \leq 2\sin^2(x + \frac{\pi}{4}) - 1 \leq 1 \rightarrow R_f = [-1, 1]$

ب) $g(x) = 3\cos^2 x - 4$

$-1 \leq \cos^2 x \leq 1 \rightarrow -3 \leq 3\cos^2 x \leq 3 \rightarrow -3 - 4 = -7 \rightarrow 3 - 4 = -1 \rightarrow R_g = [-7, -1]$

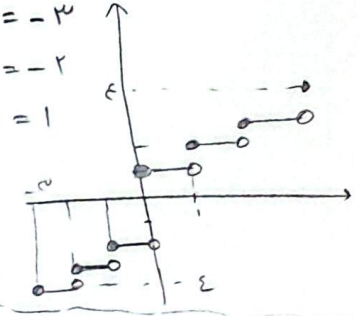
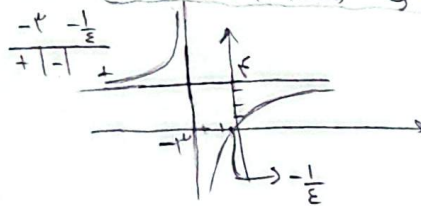
2) نادر $y = \frac{x}{|x|} + [x]$ دایره $[-3, 3]$
 $-3 \leq x < -2 \rightarrow [x] = -3 \rightarrow y = -1 - \epsilon = -\epsilon \rightarrow [-3, -2) \rightarrow y = -\epsilon$
 $-2 \leq x < -1 \rightarrow [x] = -2 \rightarrow y = -3$
 $-1 \leq x < 0 \rightarrow [x] = -1 \rightarrow y = -2$
 $0 \leq x < 1 \rightarrow [x] = 0 \rightarrow y = 1$
 $1 \leq x < 2 \rightarrow y = 2$
 $2 \leq x < 3 \rightarrow y = 3$
 $x = 3 \rightarrow y = 4$

$\frac{x}{|x|} \rightarrow -1 \quad x < 0$
 $\rightarrow 1 \quad x > 0$

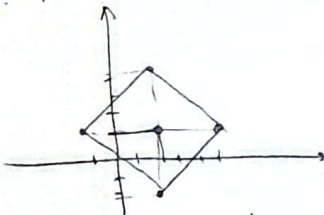
3) نادر $y = \left| \frac{\epsilon x + 1}{x + 3} \right|$

$x + 3 \neq 0 \rightarrow x \neq -3$

$\epsilon x + 1 = 0 \rightarrow x = -\frac{1}{\epsilon}$



4) نادر $|x-2| + |y-1| = 3$
 دایره مربع



ا) الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \rightarrow t + 2\sqrt{t} \rightarrow t \rightarrow [2, +\infty) \xrightarrow{\min} 2 + 2\sqrt{2} \rightarrow [2 + 2\sqrt{2}, +\infty)$

$x + \frac{1}{x} \geq 0 \rightarrow \frac{x^2 + 1}{x} \geq 0$
 $x > 0 \rightarrow x + \frac{1}{x} \geq 2$
 $t \geq 2$

ب) $y = (3 - \sin x)(1 + \sin x) \rightarrow 3 + 3\sin x - \sin x - \sin^2 x$

$- \sin^2 x + 2\sin x + 3$

$\sin x \rightarrow [-1, 1]$

max $-1 + 2 + 3 = 4$
 min $-(1) + 2(-1) + 3 = 0$
 $-\frac{b}{2a} = -\frac{2}{-1} = 2 \rightarrow \leq$

$[0, 4]$

4) $f(x) = -\frac{1}{x} + 1 \rightarrow D_f = [1, 9] \rightarrow R_f = ?$

$f(1) = 1 - \frac{1}{1} = \frac{19}{10}$

$f(2) = 1 - \frac{1}{2} = \frac{19}{20}$

$\rightarrow \frac{19}{10} + \frac{19}{20} + \dots + \frac{19}{90} = \frac{19 \times 8}{10} = 15.2$

$\frac{1}{x} (19 + 18 + \dots + 11) \Rightarrow \frac{19 \times 8}{10} = 15.2$

$f(9) = \dots - \frac{1}{9} = 1$

$$v) f(x) = \sqrt{ax^2 + bx + c} \rightarrow [r, +\infty) \xrightarrow{x=r} \varepsilon a + r^2 b + c = 0 \rightarrow$$

$$f(r) = 1$$

$$Rg = ? \rightarrow g(x) = \sqrt{bx^2 + ax - rc}$$

$$\begin{cases} b = -\varepsilon a \\ c = \varepsilon a \end{cases}$$

$$f(r) = 1 \rightarrow 1 = \sqrt{a(r-r)^2} \rightarrow 1 = \sqrt{a} \rightarrow a = 1$$

$$\begin{cases} a = 1 \\ b = -\varepsilon \\ c = \varepsilon \end{cases} \rightarrow g(x) = \sqrt{-\varepsilon x^2 + x - \varepsilon} \rightarrow -\varepsilon x^2 + x - \varepsilon \geq 0 \rightarrow \Delta < 0$$

$$Dg = \emptyset$$

$$n) f(x) = \frac{x(x-r)(x+r)}{x(x^2-\varepsilon)} \quad D_f = \mathbb{R} - \{a, b\}$$

$$R_f = [c, +\infty) - \{1\}$$

$$f\left(\frac{a+b}{rc}\right) = \frac{r-r}{r(-1)} = \frac{r}{r} = 1$$

$$f(r) = \frac{r^3 - \varepsilon(r)}{r+r} = \frac{r^3 - \varepsilon(r)}{2+r} = \frac{0}{2+r} = 0$$

$$f(x) = x(x-r) = x^2 - rx \xrightarrow{\text{تكملة المربع}} (x-\frac{r}{2})^2 - \frac{r^2}{4}$$

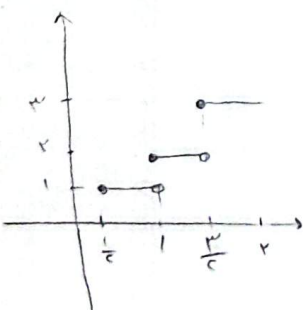
$$(x-\frac{r}{2})^2 \geq 0 \rightarrow f(x) \geq -\frac{r^2}{4} \xrightarrow{\min c} c = -\frac{r^2}{4}$$

$$f(-r) = (-r)^2 - r(-r) = r \rightarrow R_f = [-1, +\infty) - \{1\}$$

$$q) y = \sqrt[3]{x^3 + ax^2 + bx - 1} \rightarrow D_y = [a, b] \rightarrow R_y = ? \quad \underline{R_0^2}$$

$$D_y = \mathbb{R} \rightarrow R_y =$$

دالة راسيعة من $\mathbb{R}$ إلى $\mathbb{R}$ حيث a, b, c أعداد حقيقية، هل هي دالة راسيعة؟



$$1. \ y = [x] + [x + \frac{1}{r}] \rightarrow D_f = [\frac{1}{r}, r) \quad \underline{\underline{\frac{1}{r} \leq x < r}}$$

$$\frac{1}{r} \leq x < 1 \rightarrow [x] = 0$$

$$1 \leq x + \frac{1}{r} < \frac{r}{r} \rightarrow [x + \frac{1}{r}] = 1 \rightarrow y = 1$$

$$1 \leq x < \frac{r}{r} \rightarrow [x] = 1 \rightarrow [x + \frac{1}{r}] = 1 \rightarrow y = 2$$

$$\frac{r}{r} \leq x < r \rightarrow [x] = 1 \rightarrow [x + \frac{1}{r}] = 2 \rightarrow y = 3$$