

سوال ۱

$$D_{f(x)} = [-2, 4]$$

$$D_{f\left(\frac{x+2}{5}\right)} = ?$$

$$-2 \leq \frac{x+2}{5} \leq 4 \xrightarrow{\times 5} -10 \leq x+2 \leq 20 \xrightarrow{-2} -12 \leq x \leq 18 \xrightarrow{\div 3} -4 \leq x \leq 6$$

$$\Rightarrow D_{f\left(\frac{x+2}{5}\right)} = [-4, 6]$$

$$D_{f(x-2)} = [-2, 3]$$

$$D_{f(x)} = ?$$

$$-2 \leq x \leq 3 \xrightarrow{x+2} -4 \leq x+2 \leq 5 \xrightarrow{-2} -6 \leq x-2 \leq 3$$

$$\Rightarrow D_{f(x)} = [-6, 3]$$

سوال ۲

$$D_{f\left(\frac{x+1}{4}\right)} = [-1, 2]$$

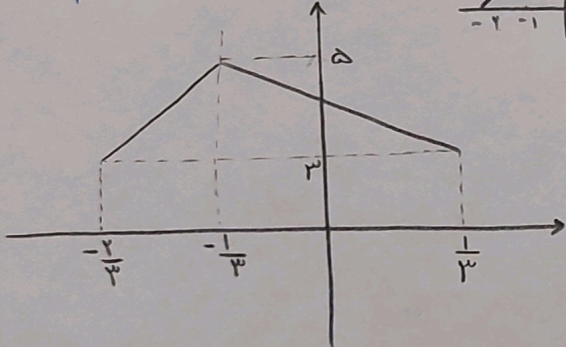
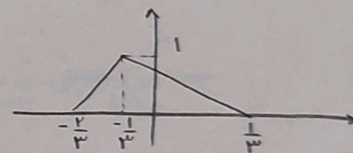
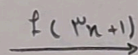
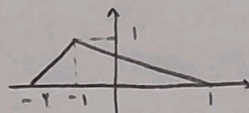
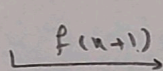
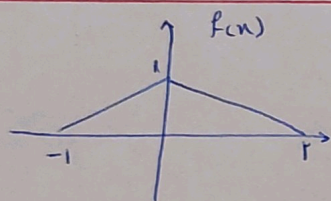
$$D_{f(2-5x)} = ?$$

$$-1 \leq x \leq 2 \xrightarrow{x+1} 0 \leq x+1 \leq 3 \xrightarrow{\div 4} 0 \leq \frac{x+1}{4} \leq \frac{3}{4} \xrightarrow{\div 2} -1 \leq \frac{x+1}{2} \leq \frac{3}{2}$$

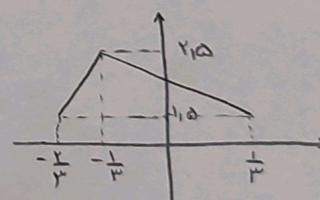
$$\Rightarrow -1 \leq 2-5x \leq \frac{3}{2} \xrightarrow{-2} -3 \leq -5x \leq -\frac{1}{2} \xrightarrow{\div -5} \frac{3}{5} \geq x \geq -\frac{1}{5} \Rightarrow D_{f(2-5x)} = \left(-\frac{1}{5}, \frac{3}{5}\right]$$

سوال ۳

$$y = 2f(x+1) + 3$$



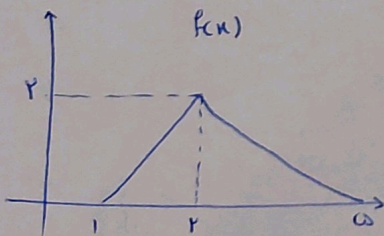
$$2f(x+1) + 3$$



$$f(x+1) + \frac{3}{2}$$

سوال ۴

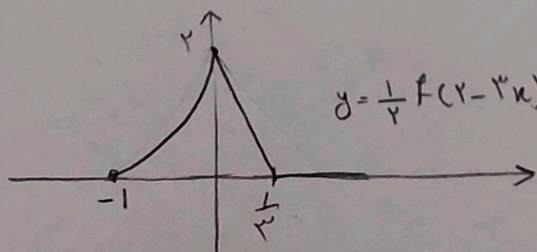
$$y = \frac{1}{4}f(2-3x)$$



$$\left| \frac{2}{2} \right| \xrightarrow{\text{مقلوب}} \left| \frac{1}{2} \right|$$

$$1 \leq 2-3x \leq 3 \xrightarrow{-2} -1 \leq -3x \leq 1 \xrightarrow{\div -3} \frac{1}{3} \geq x \geq -1$$

$\Rightarrow$



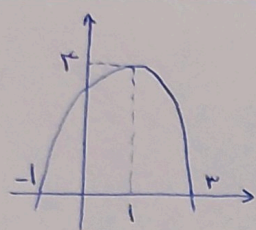
$$y = \frac{1}{4}f(2-3x)$$

$$y = -\frac{1}{3}x^2 + \frac{2}{3}x + 1 \quad \xrightarrow{\text{رأس}} (1, 3)$$

نمودار جدید را با توجه به انتقال داریم ضرب
 رأسی آن می‌نویسیم $\Rightarrow \frac{1}{3}(x-1)^2 + 3 = 0 \Rightarrow \frac{1}{3}(x-1)^2 = -3 \Rightarrow (x-1)^2 = -9 \Rightarrow x-1 = \pm 3$
 $\Rightarrow x = 4$ و $x = -2$

$$\Rightarrow \frac{1}{3}(x-1)^2 = -3 \Rightarrow (x-1)^2 = -9 \Rightarrow x-1 = \pm 3$$

$$\Rightarrow x = 4 \text{ و } -2$$



$$y = 1 - 2f(3x+2) \quad \xrightarrow{\text{رأس}} A(a, b)$$

$$ab = ?$$

$$\text{رأس} = \left| \frac{1}{3} \right| \quad \xrightarrow{\text{انتقال با ضربه}} \quad \left| \frac{a}{b} \right| = \left| \frac{\frac{1-2}{3}}{1-2} \right| = \left| \frac{\frac{-1}{3}}{-1} \right| = \frac{1}{3}$$

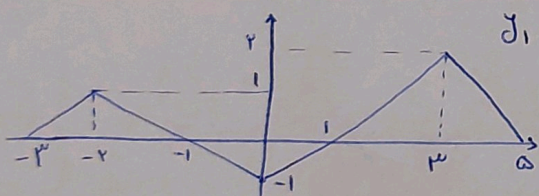
$$\Rightarrow ab = \frac{1}{3} \times -9 = -3$$

$$D_f = (a, 4] \quad D_g = [-1, b) \quad y = f(x-1) + g(x+1) \rightarrow D_y = (-3, 5) \quad a-b = ?$$

$$D_f : (a, 4] \rightarrow a < x-1 \leq 4 \xrightarrow{+1} a+1 < x \leq 5 \rightarrow D_{f(x-1)} = (a+1, 5]$$

$$D_g : [-1, b) \rightarrow -1 \leq x+1 < b \xrightarrow{-1} -2 \leq x < b-1 \rightarrow D_{g(x+1)} = [-2, b-1)$$

$$(a+1, 5] \cap [-2, b-1) = (-2, 5) \Rightarrow \begin{cases} a+1 = -2 \rightarrow a = -3 \\ b-1 = 5 \rightarrow b = 6 \end{cases} \Rightarrow a-b = -3-6 = -9$$



$$y_1 = f\left(\frac{x}{2} + 1\right)$$

$$y_2 = \sqrt{|2f(x) - 3|}$$

$$\rightarrow D_{y_1} = [-3, 5] : -3 \leq \frac{x}{2} + 1 \leq 5 \xrightarrow{-1} -4 \leq \frac{x}{2} \leq 4 \xrightarrow{\times 2} -8 \leq x \leq 8$$

$$D_{f(x)} = [-8, 8] \Leftarrow -8 \leq x \leq 8$$

$$\rightarrow R_{y_1} = [-1, 2] \rightarrow R_{f(x)} = [-1, 2]$$

$$R_{f(x)} : [-1, 2] \rightarrow -1 \leq f(x) \leq 2 \xrightarrow{\times 2} -2 \leq 2f(x) \leq 4 \xrightarrow{-3} -5 \leq 2f(x) - 3 \leq 1$$

$$\xrightarrow{\sqrt{\quad}} 0 \leq \sqrt{|2f(x) - 3|} \leq 1 \Rightarrow D_{y_2} = [-1, 1]$$

$$R_{y_2} = [0, \sqrt{5}]$$

$$f(x) = \left| \frac{x+1}{2x+1} \right| \rightarrow \text{رأسی را به دست می‌آوریم} \rightarrow g(x)$$

$$n + g(x) < 0 \quad ?$$

$$f(x) = \left| \frac{x+1}{2x+1} \right| \xrightarrow{\text{رأسی را به دست می‌آوریم}} g(x) = f(x-2) + 1 = \left| \frac{(x-2)+1}{2(x-2)+1} \right| + 1 \rightarrow g(x) = \left| \frac{x-1}{2x-3} \right| + 1$$

$$n + g(x) < 0 \rightarrow n + \left| \frac{x-1}{2x-3} \right| + 1 < 0 \rightarrow (n+1) + \left| \frac{x-1}{2x-3} \right| < 0 \Rightarrow n+1 < 0 \rightarrow n < -1$$

$$\Rightarrow (1+n) + \frac{n-1}{2n-3} < 0 \rightarrow \frac{2n^2 - 4}{2n-3} < 0 \Rightarrow 2n^2 - 4 < 0 \Rightarrow n^2 < 2 \Rightarrow n \in (-\sqrt{2}, \sqrt{2})$$