

July 19

الف) $g^p = pa^p + p - 1$, $g_1^p = pa_1^p + p - 1$, $g_p^p = pa_p^p + p - 1$ -1
 $g_1^p = g_p^p \Rightarrow g_1 = g_p$ (مطلوب)

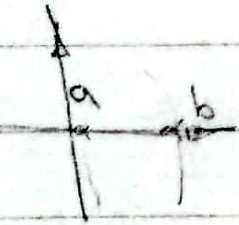
ب) $x \cos y = y \cos x$, $x=0 \rightarrow \mathbb{R}_y = \mathbb{R}$ 
 $y \in \mathbb{R}$

ج) $x+y = \sqrt{xy}$, $\sqrt{xy} = x+y \Rightarrow x+y - \sqrt{xy} = 0 \Rightarrow (\sqrt{x} - \sqrt{y})^2 = 0$
 $\Rightarrow \sqrt{x} = \sqrt{y} \Rightarrow x=y$ (مطلوب)

د) $xy^p + pa - y^p - py = 0$, $x(y^p + p) = y^p + py \Rightarrow x(y^p + p) = y(y^p + p) \Rightarrow x=y$ (مطلوب)

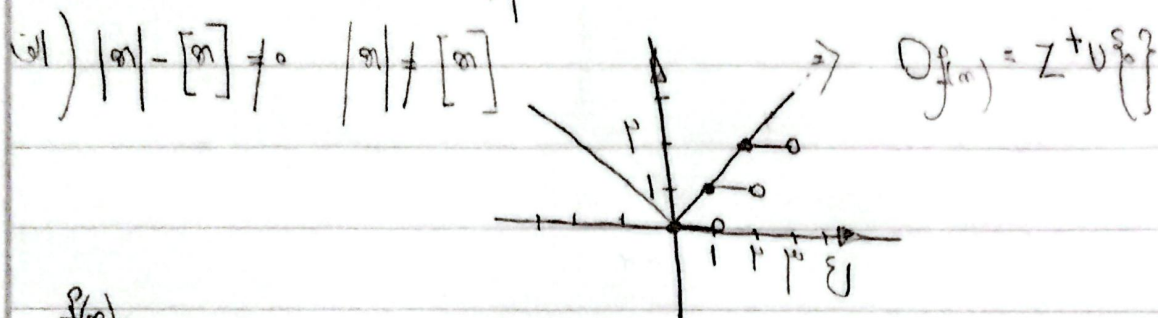
$pa^p - b$, $|a-1| \gg p \Rightarrow a-1 \gg p$, $a \gg p \Rightarrow a = -1 \rightarrow p-b \Rightarrow p-b-a-p$
 $a-1 \ll -p$, $a \ll -1 \rightarrow a-p \Rightarrow p-b-a-p$, $a+b=p$

$f(a) \sqrt{a-ba-a^p} = \sqrt{-(a^p + ba - a)}$, $-a^p + a - ba = 0$, $-a + 1 - b = 0 \Rightarrow a = 1-b$ -p
 $-b^p + a - b^p = 0 \Rightarrow b^p = a$

 $b \in (a, \infty)$, $\Rightarrow pb^p = 1-b \Rightarrow pb^p + b - 1 = 0$
 $\Rightarrow b^p \in (a, \infty)$, $b = \frac{-1 \pm \sqrt{1+1}}{p} = \frac{-1 \pm p}{p} = \frac{-1}{p}$

$\Rightarrow b^p + 1 - pb = 0 \Rightarrow b = -1 \Rightarrow a+b = 0$, $a+b =$
 $\Rightarrow b = \frac{1}{p} \Rightarrow \frac{1}{p} + \frac{1}{p} = \frac{2}{p}$, $[b, a] \Rightarrow b < a \Rightarrow a = p$, $[-1, p] \Rightarrow p-1=1$
 $a = 1 - \frac{1}{p} = \frac{p-1}{p}$

$$\begin{aligned}
 & a^2 + b^2 + c^2 = 0 \quad a=1 \Rightarrow 1 + b^2 + c^2 = 0 \Rightarrow b^2 + c^2 = -1 \quad \text{No} \\
 & \text{Df} = \begin{pmatrix} -\infty & 1 \end{pmatrix} \Rightarrow a < 0 \\
 & \Delta = b^2 - 4ac = 0 \\
 & \Rightarrow -b^2 = 0 \Rightarrow b = 0 \Rightarrow a = -b^2 = 0 \\
 & \text{Df} = \begin{pmatrix} 1 & 1 \end{pmatrix} \Rightarrow a > 0 \\
 & \Delta = b^2 - 4ac = 0 \Rightarrow b^2 = 4ac \Rightarrow b = \pm 2\sqrt{ac}
 \end{aligned}$$



iii) $\sqrt{x} + \sqrt{y} = 9$

$$\begin{aligned}
 & \sqrt{x} \geq 0 \\
 & \sqrt{y} = 9 - \sqrt{x} \Rightarrow 9 - \sqrt{x} \geq 0 \Rightarrow \sqrt{x} \leq 9 \Rightarrow x \leq 81
 \end{aligned}$$

iv) $\log \frac{x-1}{x}$

$$\begin{aligned}
 & \frac{x-1}{x} > 0 \Rightarrow x > 1 \Rightarrow \log \frac{x-1}{x} > 0 \\
 & \frac{x-1}{x} > 1 \Rightarrow x > 2 \Rightarrow \log \frac{x-1}{x} > 0
 \end{aligned}$$

$\Rightarrow \text{Df} = \{x \mid x > 2\}$

v) $\frac{1}{4 + \sqrt{|x|} - |x|}$

$$\begin{aligned}
 & 4 + \sqrt{|x|} - |x| \neq 0 \\
 & \sqrt{|x|} - |x| \neq -4
 \end{aligned}$$

$$\begin{aligned}
 & |x| = t \Rightarrow 4 + t - t^2 \neq 0 \Rightarrow (t-1)(t+4) \neq 0 \Rightarrow t \neq 1, -4 \\
 & t^2 - t - 4 \neq 0 \Rightarrow \sqrt{|x|} \neq \frac{1 \pm \sqrt{17}}{2} \Rightarrow x \neq \left(\frac{1 \pm \sqrt{17}}{2}\right)^2 \\
 & \Rightarrow \text{Df} = \mathbb{R} \setminus \left\{ \left(\frac{1 \pm \sqrt{17}}{2}\right)^2 \right\}
 \end{aligned}$$

$$g = \sqrt{\frac{p(n+p)}{n+b} + a} \quad D_f = (1, +\infty) \Rightarrow \frac{p(n+p)}{n+b} > p \Rightarrow a = -p \quad -V$$

$$\hookrightarrow n+b \neq 0 \quad n \neq -b \Rightarrow b = -p$$

$$[n] > p \Rightarrow \begin{array}{c} p \quad a \\ \hline 1 \quad p \\ \hline p(n+p) \end{array} \Rightarrow y = \sqrt{f(n) - n + 1} \quad -1$$

$$\begin{array}{l} \nearrow p \quad a \Rightarrow y = \sqrt{p(n-1) - n + 1} \Rightarrow y = \sqrt{n} \Rightarrow n > 0 \\ \searrow a \quad a \Rightarrow y = \sqrt{p(n+p) - n + 1} \Rightarrow y = \sqrt{n+1} \Rightarrow n > -1 \end{array}$$

$$\Rightarrow n > 1/2 \vee -1/2 < n < 0 \Rightarrow D_f = [-1/2, +\infty)$$

$$g = \frac{p(n^p + 1) + p^2 n^p + 1}{(p(n+p))^p} = \frac{p^2 (1 + p(n+p) + 1)}{(p(n+p))^p} = \frac{p^2 (p(n+1))^p}{(p(n+1))^p} = \frac{p^2}{p(n+1)} \quad -2$$

$$D_f = \mathbb{R} - \{-1\}$$

$$y = \sqrt{1 - \log(p(n-a))}$$

$$p(n-a) > 0 \quad p(n-a) > a \Rightarrow n > a/p \quad -1$$

$$1 - \log(p(n-a)) > 0 \Rightarrow \log(p(n-a)) < 1 \Rightarrow p(n-a) < e$$

$$p(n-a) < 1 + a \Rightarrow n < \frac{1+a}{p} \Rightarrow D_f = (a/p, \frac{1+a}{p}]$$

$$C-b = \frac{1+a}{p} - \frac{a}{p} = \frac{1}{p}$$