

الف) تعریف رافین: $y^3 = 2x^2 + 2x - 1$ $\{ \begin{matrix} \text{if } (x_1, y_1) \in f \\ \text{if } (x_2, y_2) \in f \end{matrix} \xrightarrow{\text{if } x_1 = x_2} 2x_1^2 + 2x_1 - 1 = 2x_2^2 + 2x_2 - 1 \rightarrow y_1^3 = y_2^3 \rightarrow y_1 = y_2 \rightarrow$ تابع است

ب) مثال نقض: تابع نیست
 $\text{if } x = \pi_r \rightarrow \pi_r \cos y = y \cos \pi_r \rightarrow \cos y = 0 \rightarrow y = \{ \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \}$
 $x + y - 2\sqrt{xy} = 0 \rightarrow (x - y)^2 = 0 \rightarrow x = y$ $\rightarrow$ تابع است
 ج) رسم شکل: تابع است
 د) مثال نقض: تابع نیست
 $\text{if } x = 0 \rightarrow -y^3 = 2y \rightarrow y = 0 \pm \sqrt{2}$

$f(x) \begin{cases} 2x^2 - b & |x - 1| \geq 2 \\ a + 2x & -3 \leq x \leq -1 \end{cases} \rightarrow f(x) \begin{cases} 2x^2 - b & x > 3 \text{ I} \\ 2x^2 - b & x \leq -1 \text{ II} \\ a + 2x & -3 \leq x \leq -1 \text{ III} \end{cases}$

II و III $\rightarrow 2x^2 - b = a + 2x \rightarrow 2x^2 - 2x = a + b \rightarrow$ به ازای xهای مختلف $a + b$ می تواند بیش از یک عدد باشد.
 $x = -1 \rightarrow a + b = -5$ ✓ ①
 $x = -2 \rightarrow a + b = -12$ ✗

$f(x) = \sqrt{a - bx - x^2} \rightarrow -x^2 - bx + a \geq 0 \rightarrow$
 $x_1, x_2 = \frac{b \pm \sqrt{b^2 + 4a}}{-2}$
 $a + b = \frac{b - \sqrt{b^2 + 4a}}{-2} + \frac{b + \sqrt{b^2 + 4a}}{-2} = -b$
 $a + b = -2 - 1 = -3$ ①

$ax^2 + bx + c > 0$ و $Dy = (-\infty, 2) \rightarrow$ بازه ای است و مقادیر $a=0$ بازه نیست پس $a \neq 0$
 $bx + c > 0 \rightarrow bx > -c \xrightarrow{b < 0 \text{ با توجه به اینکه}} x < -\frac{c}{b}$
 $-\frac{c}{b} = 2 \rightarrow c = -2b \rightarrow 3a + c - 3b = 3(0) + (-2b) - 3(-b) = b$ ✓

الف) $f(x) = \sqrt{\frac{1}{|x| + [x]}}$ $\rightarrow [x] + |x| \neq 0 \rightarrow |x| \neq -[x] \rightarrow$
 $\begin{cases} x \neq [x] : x > 0 \rightarrow x \notin \mathbb{Z} \\ x \neq -[x] : x < 0 \end{cases} \Rightarrow D_f = \mathbb{R} - \mathbb{Z}^{+ \cup 0}$
 $\sqrt{y} = -\sqrt{x} + 9 \rightarrow x \geq 0$ و $-\sqrt{x} + 9 \geq 0$
 $\rightarrow 9 \geq \sqrt{x} \rightarrow 81 \geq x \rightarrow D_f = [0, 81]$

$$f(x) = \sqrt{\log^2 x - 1} \quad (1) x > 0 \quad (2) x \neq 1 \quad (3) \sqrt{x-1} > 0 \rightarrow x > 1$$

$$(4) (\log^2 x - 1) > 0 \rightarrow \log^2 x - 1 \geq 1 \rightarrow x > \frac{1}{e} \rightarrow Df = \left[\frac{1}{e}, +\infty \right) \setminus \{1\}$$

$$f(x) = \log \frac{1}{\sqrt{x} - 1} \rightarrow \frac{1}{\sqrt{x} - 1} > 0 \rightarrow \sqrt{x} > 1 \rightarrow x > 1$$

$$|x| > x^2 + 3x - 12|x| \rightarrow 0 > x^2 - 13|x| + 12 \rightarrow x^2 - 13|x| + 12 > 0 \rightarrow x > 12 \text{ or } x < 1$$

$$f(x) = \sqrt{(x+a)x + ab + 1} \text{ and } Df = (-\infty, +\infty) \rightarrow$$

$$x = -\frac{a}{2} \quad (1) x + b = 0 \rightarrow x = -b \quad (2) y = 0 \rightarrow b = -\frac{a}{2} \text{ and } a = -\frac{b}{2}$$

$$f(x) = \begin{cases} \sqrt{x-1} & [x] > \varepsilon \rightarrow x > a \\ \varepsilon x + 1 & [x] \leq \varepsilon \rightarrow x < a \end{cases}$$

$$f(x) - x + 1 > 0 \rightarrow f(x) \geq x - 1 \quad f(x) = \begin{cases} \sqrt{x-1} \geq x-1 : x > a \\ \varepsilon x + 1 \geq x-1 : x < a \end{cases}$$

$$f(x) = \begin{cases} x > 0 : x > a \rightarrow x > a \quad (1) \\ x \geq -\varepsilon : x < a \rightarrow -\varepsilon \leq x < a \quad (2) \end{cases} \quad (1) \cup (2) = [-\varepsilon, +\infty)$$

$$\frac{(\varepsilon x^2 + 1)^2 + (1/x + 1)^2}{(\varepsilon x^2 + 1)^2} = \frac{1 + (1/x + 1)^2}{(\varepsilon x^2 + 1)^2} = \frac{1}{\varepsilon x^2 + 1}$$

$$Dy = \mathbb{R} - \left\{ \frac{1}{\varepsilon} \right\}$$

$$y = \sqrt{1 - (\log(x-a))} \rightarrow (1) 1 \geq \log(x-a) \rightarrow 10 \geq x-a$$

$$(2) x-a > 0 \rightarrow x > a \rightarrow 0 < x-a \leq 10 \rightarrow$$

$$\frac{a}{10} \leq x \leq \frac{10+a}{10} \rightarrow b = \frac{a}{10} \text{ and } c = \frac{10+a}{10} \rightarrow$$

$$c - b = \frac{10+a}{10} - \frac{a}{10} = \frac{10}{10} = 1$$