۱- الف) $y^3 - 3x^2 = 3x+1 \xrightarrow{\text{تعریف ریاضی}} \begin{cases} y_1^3 = 3x + 3x^2 + 1 \\ y_2^3 = 3x + 3x^2 + 1 \end{cases} \Rightarrow y_1^3 = y_2^3 \Rightarrow y_1 = y_2$

تابع است

ب) $x\cos y = y\cos x \xrightarrow{\text{مثال نقض}} x = \frac{\pi}{2} \quad \frac{\pi}{2}\cos y = y\cos\frac{\pi}{2} \Rightarrow \cos y = 0 \Rightarrow y = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$

تابع نیست

ج) $\frac{x+y}{2} = \sqrt{xy} \Rightarrow x+y = 2\sqrt{xy} \Rightarrow x+y-2\sqrt{xy} = 0 \Rightarrow (\sqrt{x}-\sqrt{y})^2 = 0$

$\sqrt{x}-\sqrt{y} = 0 \Rightarrow \sqrt{x} = \sqrt{y} \Rightarrow x = y$ تابع است

د) $xy^2 + 2x - y^3 - 2y = 0 \Rightarrow x(y^2+2) - y(y^2+2) = 0 \Rightarrow \underbrace{(y^2+2)}_{\text{حداقل ۲}}(x-y) = 0$

$\Rightarrow x - y = 0 \Rightarrow x = y$ تابع است

۲- $|x-1| \geq 2 \Rightarrow x-1 \geq 2 \Rightarrow x \geq 3$

$x-1 \leq -2 \Rightarrow x \leq -1$

$x = -1 \Rightarrow 2(-1)^2 - b = a + 2(-1) \Rightarrow 2 - b = a - 2 \Rightarrow \boxed{a+b = 4}$

۳- $y = \sqrt{a - bx - x^2} \Rightarrow -x^2 - bx + a \geq 0$

	b		a	
+	0	-	0	+

ریشه ها $a, b \Rightarrow a+b = \frac{b}{-1} \Rightarrow b = -a-b \Rightarrow -a = 2b \Rightarrow a = -2b$

$x = b \Rightarrow -b^2 - b^2 - 2b = 0 \Rightarrow -2b^2 - 2b = 0 \Rightarrow -2b(b+1) = 0$ → 0 , -1

$b = 0 \Rightarrow a = 0$ غ ق ق $\quad b = -1 \Rightarrow a = -2(-1) = 2$ ق ق $\quad \boxed{a+b = 2+(-1) = 1}$

۴- $ax^2 + bx + c > 0$ → نباید ریشه مضاعف داشته باشد $\Rightarrow a = 0$

زیرا اگر داشته باشد ۲ طرف ریشه هم علامت می شود

$bx + c > 0 \Rightarrow bx > -c \xrightarrow{b>0} x > \frac{-c}{b}$ غ ق ق → زیرا $x \in (-\infty, 2)$

$\xrightarrow{b<0} x < -\frac{c}{b} \Rightarrow \frac{-c}{b} = 2 \Rightarrow c = -2b$

$3a + c - 3|b| = -2b - 3(-b) = \boxed{b}$

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الف) $f(x)=\sqrt[3]{\dfrac{1}{|x|-[x]}}$ $\Rightarrow |x|-[x]\neq 0 \Rightarrow |x|\neq [x]$

$\Rightarrow D_f=\mathbb{R}-\{\mathbb{Z}^+\cup\{0\}\}$

ب) $f(x)=\sqrt{x}+\sqrt{y}=9 \Rightarrow x\geq 0 \quad y\geq 0 \qquad \sqrt{y}=9-\sqrt{x} \Rightarrow 9-\sqrt{x}\geq 0$

$\sqrt{x}\leq 9 \Rightarrow x\leq 81$ $\qquad D_f=[0,81]$

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الف) $f(x)=\sqrt{\log_x^{3x-1}}$ $\qquad 3x-1>0 \Rightarrow 3x>1 \Rightarrow x>\frac{1}{3}$; $x>0 \;,\; x\neq 1$ $\Rightarrow (\frac{1}{3},+\infty)-\{1\}$ I

$\log_x^{3x-1}\geq 0 \Rightarrow 3x-1\geq 1 \Rightarrow 3x\geq 2 \Rightarrow x\geq \frac{2}{3}$ II $\qquad D_f=I\cap II=[\frac{2}{3},+\infty)-\{1\}$

ب) $f(x)=\log \dfrac{1}{6+\sqrt{|x|}-|x|} \Rightarrow \dfrac{1}{6+\sqrt{|x|}-|x|}>0 \Rightarrow 6+\sqrt{|x|}-|x|>0$

$\sqrt{|x|}>|x|-6 \Rightarrow |x|>|x|^2+36-12|x| \Rightarrow |x|^2-13|x|+36<0$

$(|x|-4)(|x|-9)<0$ $\qquad$ roots: 4, 9 ; signs: $+ \; 4 \; - \; 9 \; +$ $\Rightarrow 4<|x|<9$

$\Rightarrow -9<x<9$

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$y=\sqrt{\dfrac{3x+2}{x+b}+a} \Rightarrow \dfrac{3x+2}{x+b}+a\geq 0 \Rightarrow \dfrac{3x+2+ax+ba}{x+b}\geq 0$

$x=2 \Rightarrow 2+b=0 \Rightarrow b=-2$ $\qquad$ چون دامنه $(2,+\infty]$ است پس ۲ ریشه مخرج است

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$[x] \geq 4 \rightarrow x \geq 5 \rightarrow 3x-1-x+1 \geq 0 \rightarrow x \geq 0 \rightarrow x \geq 5$

$[x] < 4 \rightarrow x < 5 \rightarrow 4x+3-x+1 \geq 0 \rightarrow 3x+4 \geq 0 \rightarrow x \geq -\frac{4}{3} \rightarrow -\frac{4}{3} \leq x < 5$

$\boxed{D_f = [-\frac{4}{3}, +\infty)}$

$y = \dfrac{24x^3+36x^2+18x+3}{(4x^2+4x+1)^2} = \dfrac{24x^3+36x^2+18x+3}{(2x+1)^4}$

$(4x^2+4x+1)^2 \downarrow (2x+1)^4$

$24x^3+36x^2+18x+3 \div (2x+1) = 12x^2+12x+3$, remainder 0

$12x^2+12x+3 \div (2x+1) = 6x+3$, remainder 0

$6x+3 \div (2x+1) = 3$, remainder 0

$= \dfrac{(2x+1)^3 \times 3}{(2x+1)^4} \rightarrow \boxed{D_f = \mathbb{R} - \{-\frac{1}{2}\}}$

۱۰

$1-\log^{2x-a} \geq 0 \rightarrow \log^{2x-a} \leq 1 \rightarrow 2x-a \leq 10 \rightarrow 2x \leq 10+a \rightarrow x \leq \frac{a+10}{2}$

$2x-a > 0 \rightarrow x > \frac{a}{2} \rightarrow x \in (\frac{a}{2}, \frac{a+10}{2}]$ ($\frac{a}{2} \downarrow b$, $\frac{a+10}{2} \downarrow c$)

$c-b = \frac{a+10}{2} - \frac{a}{2} = \frac{10}{2} = \boxed{5}$