

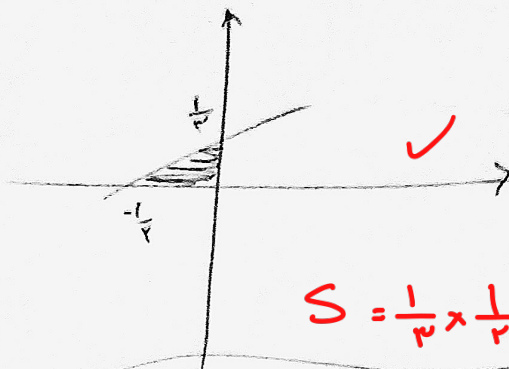
IV

مسئله ۱۵

$$y = x^2 - 1$$

$$\Rightarrow y = \frac{1}{r} x + \frac{1}{r} \Rightarrow \frac{1}{r} x + \frac{1}{r} = 0 \Rightarrow x = -\frac{1}{r}$$

۱۵



$$S = \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} = \frac{1}{r^3}$$

الف) $y = x^2 - 2x + 3$, $x \in [1, +\infty) \Rightarrow y = x^2 - 2x + 1 + 2$
 $(x-1)^2$

۲ سوال

$$\Rightarrow y - 2 = (x-1)^2 \Rightarrow \pm \sqrt{y-2} = x-1 \xrightarrow{\text{موجب } x \geq 1} y = \pm \sqrt{x-2} + 1$$

$$\Rightarrow f^{-1} = \sqrt{x-2} + 1 \text{ , } x \in [2, +\infty) \checkmark$$

چون دنیای f برداشت f^{-1} برداشت $[1, +\infty)$ است

ب) $y = x^2 - 2x + 3$, $x \in [2, +\infty)$ $R_{f^{-1}} = [2, +\infty)$

$$\Rightarrow \sqrt{x-2} + 1 \geq 3 \Rightarrow \sqrt{x-2} \geq 2 \Rightarrow x-2 \geq 4 \Rightarrow x \geq 6$$

$$f^{-1} = \sqrt{x-2} + 1 \text{ , } x \in [6, +\infty) \checkmark \quad R_{f^{-1}} = P_f \text{ چون}$$

۲ سوال

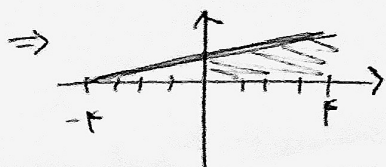
$$f(x) = 2x - \sqrt{14 - 4x(7-x)} \Rightarrow f(x) = 2x - \sqrt{14 - 14x + 4x^2}$$

$$\Rightarrow 2x - \sqrt{(2x-7)^2} = 2x - |2x-7| \xrightarrow{x \geq 7} 2x - (2x-7) = 7$$

$f^{-1} = f^{-1}$ $\swarrow$ $x < 7$ $\searrow$

$2x + 7 - 7 = 2x - 7$

$$\Rightarrow y = 2x - 7 \rightarrow y + 7 = 2x \rightarrow \frac{1}{2}x + 1 = y \xrightarrow{f^{-1}} \frac{1}{2}x + 1 < 7 \Rightarrow \frac{1}{2}x < 6 \Rightarrow x < 12$$



$$\frac{(1+2) \cdot 7}{2} = 7 \checkmark \quad x + \infty \in S$$

$$y = \frac{x}{\sqrt{x^2 - 5}} \rightarrow y_1 = y_2 \rightarrow \frac{x_1^2}{\sqrt{x_1^2 - 5}} = \frac{x_2^2}{\sqrt{x_2^2 - 5}} \quad \text{سوال 1, 175}$$

حاصل می شود

$$\Rightarrow \frac{x_1^2}{x_1^2 - 5} = \frac{x_2^2}{x_2^2 - 5} \Rightarrow x_1^2 x_2^2 - 5 x_1^2 = x_1^2 x_2^2 - 5 x_2^2$$

$$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2 \Rightarrow x_1 = x_2 \quad \checkmark \quad \text{حاصل می شود}$$

$$y = \frac{x}{\sqrt{x^2 - 5}} \Rightarrow y^2 = \frac{x^2}{x^2 - 5} \rightarrow x^2 = \frac{\Delta y^2}{y^2 - 1} \Rightarrow x = \pm \sqrt{\frac{\Delta y^2}{y^2 - 1}} \quad \text{حاصل می شود}$$

مثال $y = x \rightarrow f^{-1} = y = 0$ دقت!
توجه $\frac{|x| \sqrt{5}}{\sqrt{x^2 - 1}}$

$$g(x) = \begin{cases} x^2 - 4x + 4 & , x \geq 2 \rightarrow -\frac{b}{2a} = 2 \\ -x^2 + 4x + 4 & , x < 2 \rightarrow -\frac{b}{2a} = 2 \end{cases}$$

سوال 1, 175

1) $\rightarrow x = 2 \rightarrow 4 - 8 + 4 = 0 = k - 4 \rightarrow 1 + 2k \leq k - 4 \Rightarrow 2k \leq -5$
 2) $\rightarrow x = 3 \rightarrow -9 + 12 + 4 = 7 = k - 4 \Rightarrow k \leq -7$

مساوی هم می تونه باشه!

$$fx + |x| = y \rightarrow fx \leq x \leq 0 \Rightarrow y = fx \rightarrow x = \frac{y}{f} \quad x \geq 0 \quad \text{سوال 4}$$

$$\rightarrow fx \leq x < 0 \Rightarrow y = fx \Rightarrow x = \frac{y}{f} \quad x < 0 \quad (2)$$

$$\rightarrow f^{-1} = \begin{cases} \frac{x}{f} & x \geq 0 \\ \frac{x}{f} & x < 0 \end{cases} \rightarrow ax + b|x| \rightarrow x(a+b) \quad , x \geq 0$$

$$\rightarrow x(a-b) \quad , x < 0$$

$$\Rightarrow \begin{cases} a+b = \frac{1}{f} \\ a-b = \frac{1}{f} \end{cases} \rightarrow \begin{cases} a = \frac{f}{2} \\ b = -\frac{f}{2} \end{cases} \quad \checkmark$$

$$ab = \frac{f}{2} \times -\frac{f}{2} = -\frac{f^2}{4} \quad \checkmark$$

سوال ٩

$$(2, -10) \rightarrow \frac{f(2) + 2}{2 + m} = -10 \rightarrow \frac{10}{2 + m} = -10$$

(2)

سوال ٩

$$m = -2 \rightarrow f \circ f^{-1}(m) = m \rightarrow m + \frac{2m + 2}{m - 2} = \frac{2m^2 - 2m + 2m + 2}{m - 2} = \frac{2m^2 + 2}{m - 2}$$

$$\Rightarrow f^{-1}(m) = \frac{2m + 2}{m - 2}$$

نسبة مساوية

$$\Rightarrow \frac{2}{m} - 2m = 2 + 2 \Rightarrow m = -\frac{2}{2}$$

✓

$$\frac{a\beta + b}{c\delta + d}$$

$$f(2 - 2m) = 2 - g\left(\frac{m}{2}\right) \Rightarrow f \circ g^{-1}(2) + f^{-1}(-2) = ?$$

(2)

سوال ٩

$$\rightarrow -2 = 2 - (2) \Rightarrow \begin{cases} f(2 - 2m) = -2 \\ g\left(\frac{m}{2}\right) = 2 \end{cases} \rightarrow f\left(\frac{m}{2}\right) + 2 - 2m = 2m + 2 - 2m$$

$$= 2 \quad \checkmark$$

$$y = m + f[m] \Rightarrow m = y + f[y]$$

$$f(m) = m + f[m]$$

سوال ٩

$$[y] = [m + f[m]] \Rightarrow [y] = \frac{2}{\Delta} [x] \Rightarrow [m] = \frac{[y]}{\Delta}$$

(2)

$$\Rightarrow [y] = \frac{[m]}{\Delta} \Rightarrow 2 - f[y] = y \Rightarrow m - \frac{f[m]}{\Delta} = y$$

$$\Rightarrow g(m) = m - \frac{f[m]}{\Delta} = y \Rightarrow (f - g)(m) = m + \frac{f[m]}{\Delta} + \frac{f[m]}{\Delta}$$

$$\Rightarrow \frac{2f}{\Delta} [x]$$

سوال ٩ - صفر