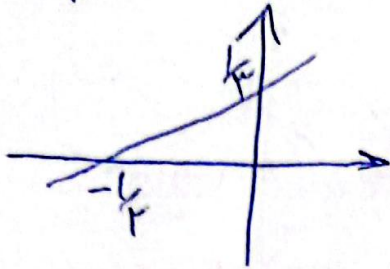


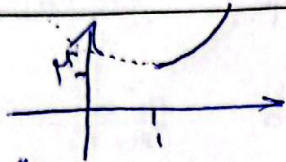
$$y = \frac{3x-1}{2} \rightarrow x = \frac{2y+1}{3} \rightarrow y^{-1} = \frac{2x+1}{3}$$



مساحت مثلث = $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12}$

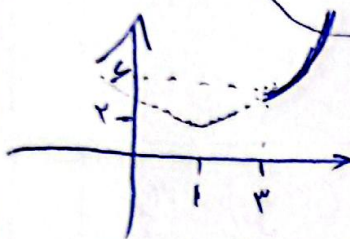
۱

$$\frac{-b}{2a} = 1$$



الف) تابع سهمی است.
 $y = (x-1)^2 + 2$ معکوس: $x = (y-1)^2 + 2$
 $y^{-1}: [2, +\infty)$ $y^{-1} = \sqrt{x-2} + 1$ (برای $x \geq 2$)

۲



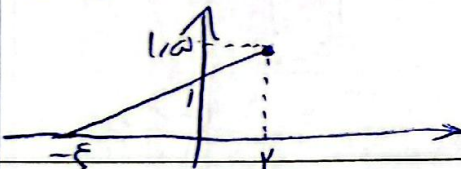
ب) یک به یک است.
 $y = (x-1)^2 + 2$ معکوس: $x = (y-1)^2 + 2$
 $y^{-1}: [2, +\infty)$ $y^{-1} = \sqrt{x-2} + 1$ (برای $x \geq 2$)

$$f(x) = 2x - \sqrt{16 - 4x(x-2)} = 2x - 2\sqrt{(x-1)^2} = 2x - 2|x-1|$$

برای $x \geq 1$: $2x - 2x + 2 = 2$ وارون پذیر

برای $x < 1$: $2x + 2x - 2 = 4x - 2$ وارون پذیر: $x = \frac{4y-2}{4} \rightarrow y^{-1} = \frac{x+2}{2}$

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برای $x < 1$: $\frac{(1+1/a)x^2}{2} = \frac{1}{1/a}$

برای یک به یک بودن: $y_1 = y_2 \rightarrow x_1 = x_2$: $\frac{x_1}{\sqrt{x_1^2 - a}} = \frac{x_2}{\sqrt{x_2^2 - a}}$ (فرض $a > 0$)

$\frac{x_1^2}{x_1^2 - a} = \frac{x_2^2}{x_2^2 - a} \rightarrow x_1^2 x_2^2 - a x_1^2 = x_1^2 x_2^2 - a x_2^2 \rightarrow x_1 = \pm x_2$

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معکوس: $x = \frac{y}{\sqrt{y^2 - a}}$ (فرض $a > 0$)
 $y^{-1} = \frac{\sqrt{a x^2}}{\sqrt{x^2 - 1}}$
 $x^2 = \frac{y^2}{y^2 - a} \rightarrow x^2 y^2 - a x^2 = y^2 \rightarrow y^2 = \frac{a x^2}{x^2 - 1}$

$g(x) \begin{cases} (x-2)^2 - 4 + k & x \geq 2 \\ -(x-3)^2 + 9 + 3k & x < 2 \end{cases} \Rightarrow x=2 \rightarrow -4+k \geq -1+9+3k$

$\rightarrow k \leq -12$

$k \leq -6$

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$$f(x) = 3x + |x| \xrightarrow{x \geq 0} \varepsilon x = y \text{ معكوس: } y^{-1} = \frac{x}{\varepsilon}$$

$$\xrightarrow{x < 0} yx = y \text{ معكوس: } y^{-1} = \frac{x}{\varepsilon}$$

$$g(x) = ax + b|x| \xrightarrow{x \geq 0} (a+b)x = \frac{x}{\varepsilon} \rightarrow a+b = \frac{1}{\varepsilon} \Rightarrow 2a = \frac{1}{\varepsilon}$$

$$\xrightarrow{x < 0} (a-b)x = \frac{x}{\varepsilon} \rightarrow a-b = \frac{1}{\varepsilon}$$

$$\rightarrow a = \frac{1}{2\varepsilon} \rightarrow b = -\frac{1}{2\varepsilon} \quad ab = \frac{1}{\varepsilon} \times -\frac{1}{\varepsilon} = -\frac{1}{\varepsilon^2}$$

$$f(x) = -10 \rightarrow \frac{x+y}{x+y} = -10 \rightarrow m = -3 \rightarrow f(x) = \frac{3x+5}{x-3}$$

$$a+d=0 \rightarrow f=f^{-1} \rightarrow f^{-1}(x) = \frac{3x+5}{x-3} \quad y = f \circ f^{-1}(x) + f^{-1}(x)$$

$$= x + \frac{3x+5}{x-3} = \frac{x^2+5}{x-3} \quad \text{معكوس: } \frac{x^2+5}{x-3} = x \rightarrow$$

$$x^2 - 3x = x^2 + 5 \rightarrow 3x + 5 = 0 \rightarrow x = -\frac{5}{3}$$

$$f(3-2x) = 2g(g(x)) = y \rightarrow 3-2x = f^{-1}(y) \text{ و } g^{-1}(3-y) = \frac{x}{\varepsilon}$$

$$\rightarrow \frac{3}{\varepsilon} - \frac{1}{\varepsilon} f^{-1}(y) = 2g^{-1}(3-y) \xrightarrow{y=-2} \frac{3}{\varepsilon} - \frac{1}{\varepsilon} f^{-1}(-2) = 2g^{-1}(\varepsilon)$$

$$\frac{3}{\varepsilon} - \frac{1}{\varepsilon} f^{-1}(-2) = 2g^{-1}(\varepsilon) \xrightarrow{\times \varepsilon} 3 - f^{-1}(-2) = \varepsilon g^{-1}(\varepsilon) \rightarrow$$

$$\varepsilon g^{-1}(\varepsilon) + f^{-1}(-2) = 3 - f^{-1}(-2) + f^{-1}(-2) = 3$$

$$y = \frac{5x-13}{1-x} \xrightarrow{\text{معكوس}} \frac{5x+13}{x+1} \xrightarrow{\text{معكوس}} \frac{5(x-2)+13}{x-2+1} - 3 = \frac{2x+5}{x-1} = f(x)$$

$$\text{معكوس: } x = \frac{5+2y}{-1+y} \rightarrow y = \frac{x+5}{x-2} = f^{-1}(x)$$

$$\text{معكوس: } f^{-1}(x) = f(x) \rightarrow \frac{x+5}{x-2} = \frac{2x+5}{x-1} \rightarrow x^2 - 3x - 5 = 0$$

$$\rightarrow x_1 = \frac{3+\sqrt{9+20}}{2} \Rightarrow x_1 + x_2 = \frac{3+\sqrt{29}}{2} + \frac{3-\sqrt{29}}{2} = \frac{6}{2} = 3$$

$$y = x + \varepsilon[x] \text{ معكوس: } x = y + \varepsilon[y] \rightarrow y = x - \varepsilon[y]$$

$$[y]: y = x - \varepsilon[y] \xrightarrow{\text{عكس}} [y] = [x - \varepsilon[y]] \rightarrow [y] = \frac{[x]}{\omega}$$

$$y = x - \varepsilon[y] \rightarrow y = x - \frac{\varepsilon}{\omega} [x] = g(x)$$

$$(f-g)(x) = x + \varepsilon[x] - x + \frac{\varepsilon}{\omega} [x] = \frac{\varepsilon}{\omega} [x]$$