۱۹,۵

تکلیف فیروزنیا

۱. ۲

$2y = 3x - 1 \rightarrow y = \frac{3}{2}x - \frac{1}{2} \xrightarrow{\text{معکوس}} x = \frac{3}{2}y - \frac{1}{2}$

$x + \frac{1}{2} = \frac{3}{2}y \rightarrow \frac{2}{3}x + \frac{1}{3} = y \xrightarrow{\text{برخورد با محور y}} \frac{1}{3}$

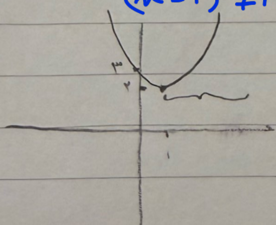
برخورد با محور x $\rightarrow 0 = \frac{2}{3}x + \frac{1}{3} \rightarrow \frac{2}{3}x = -\frac{1}{3} \rightarrow x = -\frac{1}{2}$

مساحت $= \dfrac{\frac{1}{3} \times \frac{1}{2}}{2} = \frac{1}{12}$ ✓

۲. یک به یک است ✓ الف) $y = x^2 - 2x + 3 \,,\; x \in [1, +\infty)$

۱,۵

$(x-1)^2 + 2 \longrightarrow f^{-1}(x) = \sqrt{x-2} + 1$



$\frac{-b}{2a} = \frac{2}{2} = 1 \rightarrow 1 - 2 + 3 = 2 \rightarrow R_f = [2, +\infty)$

$f^{-1}(x) = \rightarrow x = y^2 - 2y + 3 + 1 - 1 \rightarrow x = (y-2)^2 - 1$

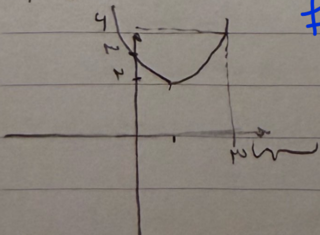
$\rightarrow (y-2)^2 = x + 1 \rightarrow y - 2 = \sqrt{x+1} \rightarrow \cancel{y = \sqrt{x+1} + 2}$

$x \geq 2$

$D_{f^{-1}} = [2, +\infty)$ ✓

یک به یک است ✓ ب) $y = x^2 - 2x + 3 \,,\; x \in [3, +\infty)$

$f^{-1}(x) = \sqrt{x-2} + 1$



$\frac{-b}{2a} = \frac{2}{2} = 1 \rightarrow 1 - 2 + 3 = 2$

$x = 3 \rightarrow 9 - 6 + 3 = 6 \quad R_f = [2, +\infty)$

$\rightarrow f^{-1}(x) = x = y^2 - 2y + 3 + 1 - 1 \rightarrow$

$x = (y-2)^2 - 1 \rightarrow x + 1 = (y-2)^2 \rightarrow y - 2 = \sqrt{x+1}$

$\cancel{y = \sqrt{x+1} + 2} \rightarrow \underbrace{x \geq 6} \rightarrow D_{f^{-1}} = [6, +\infty)$ ✓

Date　　　　　　　　　　No

آیلا عنبرخدایا

۳. ۲

$$16 + 4x^2 - 16x \rightarrow 4x^2 - 16x + 16 = 4(x-2)^2$$

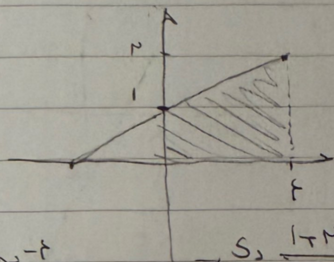
$$\sqrt{16 - 4x(4-x)} = 2|x-2| \rightarrow f(x) = 2x - 2|x-2|$$

$$\begin{array}{c|c} & 2 \\ \hline \underbrace{4x-4}_{\text{دارای بیشینه}} & \underbrace{4}_{\text{بدون بیشینه}} \end{array} \rightarrow$$

$$x = 2 \rightarrow \boxed{y = 4}$$

$$x = 4y - 4 \rightarrow x = 4(y-1) \rightarrow y - 1 = \frac{x}{4}$$

$$y = \frac{1}{4}x + 1 \rightarrow 0 = \frac{1}{4}x + 1 \rightarrow -1 = \frac{1}{4}x \rightarrow x = -4$$



$$\rightarrow S = \frac{1+2}{2} \times 4 = \underline{6}$$

۴. ۲

$$y = \frac{x}{\underset{\substack{\downarrow \\ \text{همواره} +}}{\sqrt{x^2-5}}} \xrightarrow[\text{مماس}]{\text{شیب}} (x_1, y_1) \rightarrow y_1 = \frac{x_1}{\sqrt{x_1^2-5}}$$

$$(x_2, y_2) = y_2 = \frac{x_2}{\sqrt{x_2^2-5}}$$

$$\underline{y_1 = y_2} \quad \frac{x_1}{\sqrt{x_1^2-5}} = \frac{x_2}{\sqrt{x_2^2-5}}$$

$$\rightarrow \frac{x_1^2}{x_1^2-5} = \frac{x_2^2}{x_2^2-5} \rightarrow \cancel{x_1^2 x_2^2} - 5x_1^2 = \cancel{x_2^2 x_1^2} - 5x_2^2 \rightarrow 5x_2^2 = 5x_1^2$$

$$x_1 = \pm x_2 \xrightarrow[\text{هم علامت هستند.}]{\text{ولی چون } x \text{ و } y} x_1 = x_2$$

$$y^2 = \frac{x^2}{x^2-5} \rightarrow x^2 = \frac{5y^2}{y^2-1} \rightarrow x = \pm\sqrt{\frac{5y^2}{y^2-1}} \xrightarrow[\text{هم علامت}]{y \text{ و } x} x = \sqrt{\frac{5y^2}{y^2-1}}$$

$$y = \sqrt{\frac{5x^2}{x^2-1}}$$

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آیلا فرحنتا

۵ - ۲

$g(x) \to x^2 - 4x + k \quad x \geq 2 \quad \left(\frac{-b}{2a} = \frac{4}{2} = 2\right) \to 4 - 8 + k = k - 4$

$ \to -x^2 + 6x + 3k \quad x < 2 \quad \left(\frac{-b}{-2a} = \frac{-6}{-2} = 3\right) \to$ عرض نقطه شکست

$x = 2 \to -4 + 12 + 3k \to 3k + 8$

$3k + 8 \leq k - 4 \to 2k \leq -12 \to k \leq -6$

آیدا منفردی‌نیا

۶. ۲

$f(x)=3x+|x| \longrightarrow$ (0)

$x=3y-y \qquad x=3y+y$

$x=2y \qquad x=4y$

$y=\frac{x}{2}=\frac{2x}{4} \qquad y=\frac{x}{4}$

$\cancel{x}(a+b)=\frac{1}{4}\cancel{x} \to a+b=\frac{1}{4}$

$\cancel{x}(a-b)=\frac{2}{4}\cancel{x} \to a-b=\frac{2}{4} \longrightarrow 2a=\frac{3}{4} \to a=\frac{3}{8}$ ✓ $\mid$ $b=\frac{-1}{8}$ ✓

$ab=\frac{3}{8}\times\frac{-1}{8}=\frac{-3}{64}$ ✓

۷. ۲

$f(x)=\frac{3x+2}{x+m} \to \frac{\overset{10}{\cancel{6+2}}}{\underbrace{2+m}}=-10$

$(2,-10) \qquad 2+m=-1 \to m=-3$

$f(x)=\frac{3x+2}{x-3} \quad a=d \longrightarrow f(x)=f^{-1}(x)$

$y=f\circ f^{-1}(x)+f^{-1}(x) \longrightarrow x+\frac{3x+2}{x-3}=y$

$x\neq 3$

شرط دامنه

$\to x=y \longrightarrow \cancel{x}+\frac{3x+2}{x-3}=\cancel{x} \longrightarrow 3x+2=0 \to x=\frac{-2}{3}$ ✓

۸. $f(3-2x)=t \to f^{-1}(t)=3-2x$

$2-g(\frac{x}{2})=t \to g(\frac{x}{2})=2-t \to g^{-1}(2-t)=\frac{x}{2} \to 2x=4g^{-1}(2-t)$ ۲

$f^{-1}(t)=3-2x \longrightarrow f^{-1}(t)=3-4g^{-1}(2-t) \xrightarrow{t=-2} f^{-1}(-2)=3-4g^{-1}(4)$

$\to 4g^{-1}(4)+f^{-1}(-2)=4g^{-1}(4)+3-4g^{-1}(4)=3$ ✓

آیلا فیروزنیا

9. ۲

$$y=\frac{5x-13}{1-x}\rightarrow -y=\frac{-5x-13}{1+x}\rightarrow y=\frac{5x+13}{1+x}$$

$$\xrightarrow[(x-2)]{\text{۲ واحد راست}} y=\frac{5(x-2)+13}{(x-2)+1}\xrightarrow{\text{۳ واحد پایین}}\frac{5(x-2)+13}{\underbrace{(x-2)+1}_{x-1}}-3$$

$$y=\frac{5x-10+13-3x+3}{x-1}=\frac{2x+6}{x-1}$$

$$f(x)=\frac{2x+6}{x-1}$$

$$f^{-1}(x)=\frac{x+6}{x-2}$$

$$\longrightarrow \underset{x\neq 1}{\frac{2(x+3)}{x-1}}=\underset{x\neq 2}{\frac{x+6}{x-2}}\rightarrow 2(x^2+x-6)=x^2+5x-6$$

$$\rightarrow 2x^2+2x-12=x^2+5x-6\rightarrow x^2-3x-6=0 \quad \frac{\text{مجموع}}{S}=\frac{-b}{a}=\frac{3}{1}=3$$ ✓

10. ۲

$$y=x+4[x]\longrightarrow x=y+4[y]$$

$$[x]=[y+4[y]]\longrightarrow [x]=[y]+4[y]\rightarrow [x]=5[y]$$

$$[y]=\frac{[x]}{5}\rightarrow 4[y]=\frac{4}{5}[x]$$

$$x=y+\frac{4}{5}[x]\rightarrow y=x-\frac{4}{5}[x]$$

$$f(x)=x+4[x]$$
$$g(x)=x-\frac{4}{5}[x]$$

$$\longrightarrow f(x)-g(x)=\cancel{x}+4[x]\cancel{-x}+\frac{4}{5}[x]$$

$$=\frac{24}{5}[x]$$ ✓