آیلا فیروزنیا

۱. $2y = 3x - 1 \rightarrow y = \frac{3}{2}x - \frac{1}{2} \xrightarrow{\text{معکوس}} x = \frac{3}{2}y - \frac{1}{2}$

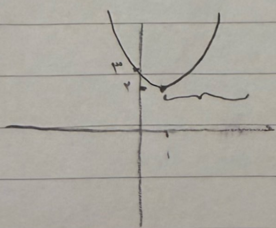
$x + \frac{1}{2} = \frac{3}{2}y \rightarrow \frac{2}{3}x + \frac{1}{3} = y$ برخورد با محور y : $\frac{1}{3}$

برخورد با محور x : $0 = \frac{2}{3}x + \frac{1}{3} \rightarrow \frac{2}{3}x = -\frac{1}{3} \rightarrow x = -\frac{1}{2}$

مساحت $= \dfrac{\frac{1}{3} \times \frac{1}{2}}{2} = \frac{1}{12}$

۲. یک به یک است

الف) $y = x^2 - 2x + 3 \;,\; x \in [1, +\infty)$



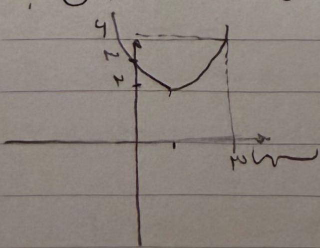
$\frac{-b}{2a} = \frac{2}{2} = 1 \rightarrow 1 - 2 + 3 = 2 \rightarrow R_f = [2, +\infty)$

$f^{-1}(x) = \rightarrow x = y^2 - 2y + 3 + 1 - 1 \rightarrow x = (y-2)^2 - 1$

$\rightarrow (y-2)^2 = x + 1 \rightarrow y - 2 = \sqrt{x+1} \rightarrow y = \sqrt{x+1} + 2$

$D_{f^{-1}} = [2, +\infty)$ $\quad x \geq 2$

ب) $y = x^2 - 2x + 3 \;,\; x \in [3, +\infty)$ یک به یک است



$\frac{-b}{2a} = \frac{2}{2} = 1 \rightarrow 1 - 2 + 3 = 2$

$x = 3 \rightarrow 9 - 6 + 3 = 6 \quad R_f = [2, +\infty)$

$\rightarrow f^{-1}(x) = x = y^2 - 2y + 3 + 1 - 1 \rightarrow$

$x = (y-2)^2 - 1 \rightarrow x + 1 = (y-2)^2 \rightarrow y - 2 = \sqrt{x+1}$

$y = \sqrt{x+1} + 2 \rightarrow \underbrace{x \geq 6}_{} \rightarrow D_{f^{-1}} = [6, +\infty)$

آیلا منیروندنیا

۳.

$$16 + 4x^2 - 16x \longrightarrow 4x^2 - 16x + 16 = 4(x-2)^2$$

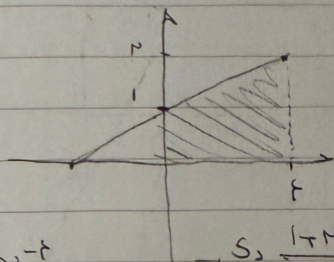
$$\sqrt{16 - 4x(4-x)} = 2|x-2| \longrightarrow f(x) = 2x - 2|x-2|$$

$$\begin{array}{c|c} & 2 \\ \hline \underbrace{4x-4}_{\text{وابسته به متغیره}} & \underbrace{4}_{\text{بدون متغیره}} \end{array}$$

$x = 2 \rightarrow \boxed{y = 4}$

$x = 4y - 4 \rightarrow x = 4(y-1) \rightarrow y - 1 = \frac{x}{4}$

$y = \frac{1}{4}x + 1 \rightarrow 0 = \frac{1}{4}x + 1 \rightarrow \frac{1}{4}x = -1 \rightarrow x = -4$



$\rightarrow S = \frac{1+2}{2} \times 4 = 6$

۴.

$y = \frac{x}{\sqrt{x^2 - 5}}$ (+ مراقبت) $\xrightarrow[\text{تماس}]{\text{شرط}}$ $(x_1, y_1) \rightarrow y_1 = \frac{x_1}{\sqrt{x_1^2 - 5}}$

$(x_2, y_2) = y_2 = \frac{x_2}{\sqrt{x_2^2 - 5}}$

$\underline{y_1 = y_2} \quad \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}}$

$$\rightarrow \frac{x_1^2}{x_1^2 - 5} = \frac{x_2^2}{x_2^2 - 5} \rightarrow \cancel{x_1^2 x_2^2} - 5x_1^2 = \cancel{x_2^2 x_1^2} - 5x_2^2 \rightarrow 5x_2^2 = 5x_1^2$$

$x_1 = \pm x_2 \xrightarrow[\text{هم علامت هستند.}]{\text{ولی چون } y \text{ و } x} x_1 = x_2$

$$y^2 = \frac{x^2}{x^2 - 5} \rightarrow x^2 = \frac{5y^2}{y^2 - 1} \rightarrow x = \pm\sqrt{\frac{5y^2}{y^2 - 1}} \xrightarrow[\text{هم علامت}]{y \text{ و } x} x = \sqrt{\frac{5y^2}{y^2 - 1}}$$

$$y = \sqrt{\frac{5x^2}{x^2 - 1}}$$

Date No

آیلا فرجنیا

۵- $g(x) \begin{cases} x^2 - 4x + k & x \geq 2 \quad \left(\frac{-b}{2a} = \frac{4}{2} = 2 \rightarrow 4 - 8 + k = k - 4\right. \\ -x^2 + 6x + 3k & x < 2 \quad \left(\frac{-b}{-2a} = \frac{-6}{-2} = 3 \rightarrow\right. \end{cases}$ در دامنه نیست

$\underbrace{x = 2 \rightarrow -4 + 12 + 3k \rightarrow 3k + 8}$

$3k + 8 \leq k - 4 \rightarrow 2k \leq -12 \rightarrow \underline{k \leq -6}$

آیدا عزیزنیا

۶. $f(x)=3x+|x| \to$ 0

$x<0$: $x=3y-y$, $x=2y$, $y=\frac{x}{2}=\frac{2x}{4}$ $\quad$ $x\geq 0$: $x=3y+y$, $x=4y$, $y=\frac{x}{4}$

$\cancel{x}(a+b)=\frac{1}{4}\cancel{x} \to a+b=\frac{1}{4}$

$\cancel{x}(a-b)=\frac{2}{4}\cancel{x} \to a-b=\frac{2}{4} \to 2a=\frac{3}{4} \to a=\frac{3}{8}$ / $b=\frac{-1}{8}$

$ab=\frac{3}{8}\times\frac{-1}{8}=\frac{-3}{64}$

۷. $f(x)=\frac{3x+4}{x+m} \to \frac{\overset{10}{\cancel{6+4}}}{\underbrace{2+m}}=-10$

$(2,-10)$ $\quad$ $2+m=-1 \to m=-3$

$f(x)=\frac{3x+4}{x-3} \to f(x)=f^{-1}(x)$

$a=d$

$y=f\circ f^{-1}(x)+f^{-1}(x) \to x+\frac{3x+4}{x-3}=y$

$x\neq 3$

شرط

$\to x=y \to y=x \to \cancel{x}+\frac{3x+4}{x-3}=\cancel{x} \to 3x+4=0 \to x=\frac{-4}{3}$

۸. $f(3-2x)=t \to f^{-1}(t)=3-2x$

$2-g(\frac{x}{2})=t \to g(\frac{x}{2})=2-t \to g^{-1}(2-t)=\frac{x}{2} \to 2x=4g^{-1}(2-t)$

$f^{-1}(t)=3-2x \to f^{-1}(t)=3-4g^{-1}(2-t) \xrightarrow{t=-2} f^{-1}(-2)=3-4g^{-1}(4)$

$\to 4g^{-1}(4)+f^{-1}(-2)=4g^{-1}(4)+3-4g^{-1}(4)=3$

آیلا فرهنیا

9. $y=\frac{5x-13}{1-x} \rightarrow -y=\frac{-5x-13}{1+x} \rightarrow y=\frac{5x+13}{1+x}$

$\xrightarrow[(x-2)]{\text{۲ واحد راست}} y=\frac{5(x-2)+13}{(x-2)+1} \xrightarrow{\text{۳ واحد پایین}} \frac{5(x-2)+13}{\underbrace{(x-2)+1}_{x-1}}-3$

$y=\frac{5x-10+13-3x+3}{x-1}=\frac{2x+6}{x-2}$

$f(x)=\frac{2x+6}{x-1}$

$f^{-1}(x)=\frac{x+6}{x-2}$

$\longrightarrow \frac{2(x+3)}{x-1}=\frac{x+6}{x-2} \rightarrow 2(x^2+x-6)=x^2+5x-6$

$x\neq 1 \qquad x\neq 2$

$2x^2+2x-12=x^2+5x-6 \rightarrow x^2-3x-6=0 \quad \frac{\text{مجموع}}{\text{رر}}=\frac{-b}{a}=\frac{3}{1}=3$

10. $y=x+4[x] \longrightarrow x=y+4[y]$

$[x]=[y+4[y]] \longrightarrow [x]=[y]+4[y] \rightarrow [x]=5[y]$

$[y]=\frac{[x]}{5} \rightarrow 4[y]=\frac{4}{5}[x]$

$x=y+\frac{4}{5}[x] \rightarrow y=x-\frac{4}{5}[x]$

$f(x)=x+4[x]$

$g(x)=x-\frac{4}{5}[x]$

$\longrightarrow f(x)-g(x)=\cancel{x}+4[x]\cancel{-x}+\frac{4}{5}[x]$

$=\frac{24}{5}[x]$