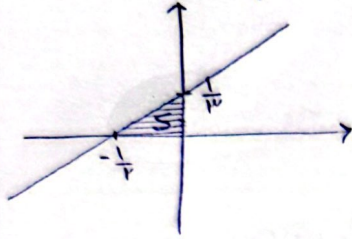


$$2y = 3x - 1 \rightarrow x = \frac{2y+1}{3} \rightsquigarrow y = \frac{2}{3}x + \frac{1}{3}$$



$$S = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$$

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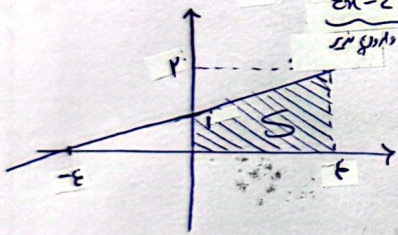
الف) $y = x^2 - 2x + 3 \rightarrow y = x^2 - 2x + 1 + 2 \rightarrow y = (x-1)^2 + 2$ $x = 1 + \sqrt{y-2} \rightsquigarrow y = 1 + \sqrt{x-2}$
 $x \in [1, +\infty)$ $\left. \begin{matrix} \text{ent} \\ | \end{matrix} \right| 2$ $(x \geq 2)$

ب) $y = x^2 - 2x + 3 \rightarrow y = x^2 - 2x + 1 + 2 \rightarrow y = (x-1)^2 + 2$ $x = 1 + \sqrt{y-2} \rightsquigarrow y = 1 + \sqrt{x-2}$
 $x \in [3, +\infty)$ $\left. \begin{matrix} \text{ent} \\ | \end{matrix} \right| 4$ $(x \geq 4)$

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$$f(x) = 2x - \sqrt{14 - 2x(2-x)} \rightarrow f(x) = 2x - \sqrt{2x^2 - 14x + 14} \rightarrow y = 2x - 2|x-2|$$

$$f(x) = \frac{2}{2x-2} = \frac{1}{x-1} \quad y = 2x - 2 \quad x = \frac{y+2}{2} \rightarrow y = \frac{1}{2}x + 1$$



$$S = \frac{(1+2) \times 2}{2} = 4$$

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$$y = \frac{x}{\sqrt{x^2-5}} \rightarrow x^2-5 > 0 \quad x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, +\infty)$$

$$y' = \frac{1}{\sqrt{x^2-5}} - \frac{x^2}{(x^2-5)^{3/2}} \rightarrow y' = \frac{(x^2-5) - x^2}{(x^2-5)^{3/2}} = \frac{-5}{(x^2-5)^{3/2}} \quad (x^2-5)^{3/2} > 0 \rightarrow y' = \frac{-5}{(x^2-5)^{3/2}} < 0$$

$x \rightarrow \sqrt{5}^+$ $y \rightarrow +\infty$ $y \in (1, +\infty)$ $x \rightarrow -\sqrt{5}^-$ $y \rightarrow -\infty$ $y \in (-\infty, -1)$
 $y = \frac{x}{\sqrt{x^2-5}} \rightarrow x\sqrt{y^2-5} = y \quad x^2y^2 - y^2 = 5x^2 \quad y = \pm \sqrt{\frac{5x^2}{x^2-1}} \quad D_{y^{-1}} \in (-\infty, -1) \cup (1, +\infty)$

۴

$$y = x^2 - 2kx + k \rightarrow \frac{-b}{2a} = \frac{2}{2} = 1 \quad y = -x^2 + 4x + 3 \rightarrow \frac{-b}{2a} = \frac{-4}{-2} = 2$$

$$x = 2 \rightarrow 2 - 4 + k = k - 2$$

$$x = 2 \rightarrow 2 - 4 + 4 + 3 = 5$$

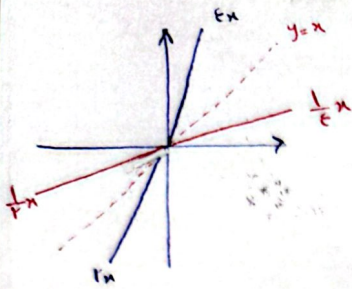
$$1 + 2k < k - 2$$

$$2k < -12$$

$$k < -6$$

۵

$$f(x) = 3x + |x|$$



$$f^{-1}(x) = \begin{cases} (a+b)x & x \geq 0 \\ (a-b)x & x < 0 \end{cases} = \begin{cases} \frac{1}{2}x & x \geq 0 \\ \frac{1}{4}x & x < 0 \end{cases}$$

$$\begin{cases} a+b = \frac{1}{2} \\ a-b = \frac{1}{4} \end{cases}$$

$$2a = \frac{3}{4} \quad a = \frac{3}{8}, \quad b = -\frac{1}{8} \Rightarrow ab = -\frac{3}{64}$$

6

$$f(x) = \frac{3x + \varepsilon}{x + m} \xrightarrow{f(r) = -1} -1 = \frac{3r + \varepsilon}{r + m} \Rightarrow r + m = -1 \quad (m = -3) \quad f(x) = \frac{3x + \varepsilon}{x - 3}$$

$$y = f \circ f^{-1}(x) + f^{-1}(x) = f(f(x)) + f(x)$$

$$f(x) = f^{-1}(x) \leftarrow \text{مساوی است}$$

$$\frac{3 \left(\frac{3x + \varepsilon}{x - 3} \right) + \varepsilon}{\left(\frac{3x + \varepsilon}{x - 3} \right) - 3} + \frac{3x + \varepsilon}{x - 3} = \frac{x + \varepsilon}{x - 3} \xrightarrow{\text{مساوی است}} x = \frac{x + \varepsilon}{x - 3} \quad x - 3x = x + \varepsilon \quad 3x = -\varepsilon \quad (x = -\frac{\varepsilon}{3})$$

7

$$f(3 - rx) = r - g\left(\frac{x}{r}\right)$$

$$f(3 - rx) = r - \varepsilon = -r$$

$$g^{-1}(\varepsilon) = \frac{x}{r} \leftarrow g\left(\frac{x}{r}\right) = f\left(\frac{x}{r}\right)$$

$$f(3 - rx) = -r \rightarrow f^{-1}(-r) = 3 - rx$$

$$\varepsilon g^{-1}(\varepsilon) + f^{-1}(-r) = \varepsilon \left(\frac{x}{r} \right) + 3 - rx = rx + 3 - rx = (3)$$

8

$$y = \frac{2x - 13}{1 - x} \rightarrow -y = \frac{-2x + 13}{1 + x} \quad -y = \frac{-2(x - r) - 13}{1 + (x - r)} - 13 = \frac{-2x - 13}{x - 1}$$

$$\frac{2x + 13}{x - 1} = \frac{-2x - 13}{1 + x}$$

$$2x^2 + 13x + 13 = -2x^2 - 13x + 13$$

$$4x^2 + 26x - 0 = 0$$

$$\alpha + \beta = S = -\frac{b}{a} = (-1, 4)$$

9

$$f(x) = x + f[x] = y \rightarrow f^{-1}(y) \rightarrow x = y + f[y] \quad y = x + f[x] \quad x = y + f[x + f[x]]$$

$$(f - g)(x) = x + f[x] - x + g[x]$$

$$x = y + f[x] + g[x]$$

$$f^{-1}(y) = g(x) = y = x - f[x]$$

$$= f[x]$$

10