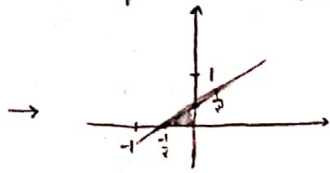


۱۴۵

$$2y = 3x - 1 \rightarrow y = \frac{3x-1}{2} \Rightarrow f(n) = \frac{3n-1}{2} \Rightarrow y = \frac{3x-1}{2} \rightarrow 3x-1 = 2y \rightarrow 3x = 2y+1$$

$$\rightarrow x = \frac{2y+1}{3} \rightarrow y = \frac{3x-1}{2} \rightarrow f^{-1}(n) = \frac{3n-1}{2} \rightarrow \begin{cases} x=0, y=\frac{1}{2} \\ x=-\frac{1}{3}, y=0 \end{cases}$$

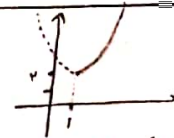


$$\rightarrow S_{\text{منطقه}} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = \boxed{\frac{1}{12}} \checkmark$$

(۲)

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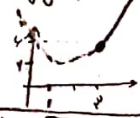
الف) $y = x^2 - 2x + 3, x \in [1, +\infty) \rightarrow \Delta < 0, \text{ent} \left| \frac{-b}{2a} = \frac{1}{2} \right|$



طبق شکل در این بازه یک به یک است $\rightarrow y = x^2 - 2x + 3 = x^2 - 2x + 1 + 2 = (x-1)^2 + 2 \rightarrow y - 2 = (x-1)^2$

$\rightarrow y - 2 = (x-1)^2 \rightarrow \sqrt{y-2} = |x-1| \xrightarrow{x \geq 1} \sqrt{y-2} = x-1 \rightarrow x = \sqrt{y-2} + 1 \rightarrow \boxed{f^{-1}(n) = \sqrt{n-2} + 1, D_{f^{-1}} = [2, +\infty)}$ ✓

ب) $y = x^2 - 2x + 3, x \in [3, +\infty) \rightarrow \Delta < 0, \text{ent} \left| \frac{1}{2} \right|$



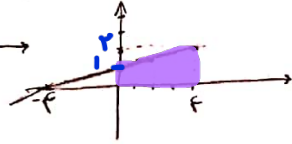
طبق شکل تابع در این بازه یک به یک است $\rightarrow y = x^2 - 2x + 3 = (x-1)^2 + 2 \rightarrow y - 2 = (x-1)^2 \rightarrow \sqrt{y-2} = |x-1| \xrightarrow{x \geq 3} \sqrt{y-2} = x-1 \rightarrow x = \sqrt{y-2} + 1 \rightarrow \boxed{f^{-1}(n) = \sqrt{n-2} + 1, R_f = D_{f^{-1}} = [4, +\infty)}$ ✓

(۲)

$f(x) = 2x - \sqrt{4 - 4x(f-x)} = 2x - \sqrt{4 - 4x(f-x)} = 2x - \sqrt{4 - 4x(f-x)} = 2x - \sqrt{4 - 4x(f-x)} = 2x - \sqrt{4 - 4x(f-x)}$

$\rightarrow f(x) = 4x - 4, x \in (-\infty, 2] \rightarrow D_f = (-\infty, 2] \rightarrow R_f = (-\infty, 4] \Rightarrow y = 4x - 4 \rightarrow 4x = y + 4 \rightarrow x = \frac{y+4}{4} \rightarrow y = \frac{4x-4}{1} \checkmark$

$\rightarrow f^{-1}(n) = \frac{n+4}{4}, D_{f^{-1}} = R_f = (-\infty, 4] \checkmark$



$S = \frac{(1+2) \cdot 4}{2} = 6$

(۱,۵)

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$y = \frac{x}{\sqrt{x^2-5}} \rightarrow \text{if } x = f(n) \text{ و } \sqrt{x^2-5} = g(n) \rightarrow y = h(n) = \frac{f(n)}{g(n)}$

این تابع یک به یک است $\rightarrow g(n) = \sqrt{x^2-5} \rightarrow h(n) = \frac{x}{\sqrt{x^2-5}} \rightarrow y = \frac{x}{\sqrt{x^2-5}} \rightarrow y^2 = \frac{x^2}{x^2-5} \rightarrow y^2(x^2-5) = x^2 \rightarrow x^2(y^2-1) = 5y^2 \rightarrow x^2 = \frac{5y^2}{y^2-1} \rightarrow x = \frac{\sqrt{5}y}{\sqrt{y^2-1}} \rightarrow f^{-1}(n) = \frac{n\sqrt{5}}{\sqrt{n^2-1}} \checkmark$

$\rightarrow x = \frac{\sqrt{5}y}{\sqrt{y^2-1}} \rightarrow y = \frac{x\sqrt{5}}{\sqrt{x^2-1}} \rightarrow f^{-1}(n) = \frac{n\sqrt{5}}{\sqrt{n^2-1}} \checkmark$

(۲)

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اگر $x=2$ باشد ضابطه در تابع باید برابر باشد تا یک به یک شود تا ضابطه در اول که چتر شود.

$\left. \begin{aligned} x=2 &\rightarrow f-1+k \\ x=2 &\rightarrow -f+12+3k \end{aligned} \right\} \rightarrow f-1+k = -f+12+3k \rightarrow k-f = 3k+11 \rightarrow 2k = -12 \rightarrow k = -6$

$\rightarrow -f+12+3k < f-1+k \rightarrow 1+3k < k-f \rightarrow 2k < -12 \rightarrow k < -6$

$\rightarrow k < -6 \rightarrow \boxed{k \in (-\infty, -6]} \checkmark$

(۲)

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$$y = 3x + |x| \quad \begin{cases} x \geq 0 \rightarrow 3x + x = 4x \rightarrow y = 4x \rightarrow x = \frac{y}{4} \rightarrow f^{-1}(y) = \frac{y}{4}, & x \geq 0 \\ x < 0 \rightarrow 3x - x = 2x \rightarrow y = 2x \rightarrow x = \frac{y}{2} \rightarrow f^{-1}(y) = \frac{y}{2}, & x < 0 \end{cases}$$

$$\rightarrow f^{-1} \cdot g(y) = a x + b |x| \quad \begin{cases} x \geq 0 \rightarrow a x + b x = \frac{x}{4} \rightarrow x(a+b) = \frac{x}{4} \rightarrow a+b = \frac{1}{4} \\ x < 0 \rightarrow a x - b x = \frac{x}{2} \rightarrow x(a-b) = \frac{x}{2} \rightarrow a-b = \frac{1}{2} \end{cases} \rightarrow \begin{cases} a+b = \frac{1}{4} \\ a-b = \frac{1}{2} \end{cases} \rightarrow \begin{cases} 2a = \frac{3}{4} \rightarrow a = \frac{3}{8} \\ b = -\frac{1}{8} \end{cases}$$

$$\Rightarrow a b = \frac{3}{8} \cdot -\frac{1}{8} = -\frac{3}{64} \quad \checkmark$$

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$$y = \frac{3x+f}{x+m} \xrightarrow{y=0} -10 = \frac{4+f}{1+m} \rightarrow -10-m = \frac{4+f}{1+m} \rightarrow -10m = -4-f \rightarrow 10m = 4+f \rightarrow m = \frac{4+f}{10} \rightarrow f(x) = \frac{3x+f}{x-\frac{4+f}{10}}$$

$$\rightarrow f^{-1}(y) : y = \frac{3x+f}{x-\frac{4+f}{10}} \xrightarrow{ax+c=0} f^{-1}(y) = f(x) = \frac{3x+f}{x-\frac{4+f}{10}} \rightarrow g = \frac{f \circ f^{-1}(y)}{x} + \frac{f^{-1}(y)}{f(x)}$$

$$\rightarrow y = x + f(x) \rightarrow x + f(x) = x \rightarrow f(x) = 0 \rightarrow \frac{3x+f}{x-\frac{4+f}{10}} = 0 \rightarrow 3x+f = 0 \rightarrow 3x+4 = 0$$

$$\rightarrow x = -\frac{4}{3} \quad \checkmark \rightarrow \text{با طول } x = -\frac{4}{3} \text{ میسر از نام اول و سوم مطابقت دارد}$$

۲۲

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$$f(3-2x) = y - g\left(\frac{x}{y}\right) \rightarrow f(3-2x) = t \quad \text{از طرفین } f^{-1} \rightarrow f^{-1} \circ f(3-2x) = f^{-1}(t) \rightarrow f^{-1}(t) = 3-2x$$

$$t = y - g\left(\frac{x}{y}\right) \rightarrow g\left(\frac{x}{y}\right) = y - t \quad \text{از طرفین } g^{-1} \rightarrow g^{-1} \circ g\left(\frac{x}{y}\right) = g^{-1}(y-t) \rightarrow g^{-1}(y-t) = \frac{x}{y} \quad u = \frac{3-f^{-1}(t)}{y}$$

$$u = 2g^{-1}(y-t)$$

$$\text{جای } x \text{ و } y \text{ را عوض کن!}$$

$$\text{if } t = -2 \rightarrow f^{-1}(-2) = 3-2(-2) = 7 \quad , \quad g^{-1}(y-(-2)) = g^{-1}(4) = \frac{4}{y} = 2$$

$$\rightarrow 4g^{-1}(4) + f^{-1}(-2) = ? = 4(2) + 7 = 15 \quad \frac{3-f^{-1}(t)}{y} = 2g^{-1}(y-t) \quad t = -2$$

$$\frac{3-f^{-1}(-2)}{y} = 4g^{-1}(4) \quad \text{جواب ۳}$$

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$$y = \frac{\delta x - 13}{1-x} \rightarrow -y = \frac{\delta(-x) - 13}{1-(-x)} = \frac{-\delta x - 13}{1+x} \leftarrow \text{از طرفین } \delta \text{ و } x \text{ را عوض کن}$$

$$\rightarrow -y = \frac{-\delta x - 13}{1+x} \rightarrow y = \frac{\delta x + 13}{x+1} \quad \text{دو طرف را با هم جمع کن} \rightarrow y = \frac{\delta(x-2) + 13}{(x-2)+1} = \frac{\delta x - 10 + 13}{x-1} = \frac{\delta x + 3}{x-1}$$

$$\xrightarrow{\text{دو طرف را با هم جمع کن}} \frac{\delta x + 3}{x-1} - 3 = \frac{\delta x + 3 - 3(x-1)}{x-1} = \frac{\delta x + 3 - 3x + 3}{x-1} = \frac{-2x + 6}{x-1} \rightarrow f(x) = y = \frac{2x+14}{x-1} \rightarrow x = -\frac{y-14}{y-2}$$

$$\rightarrow x = \frac{y+14}{y-2} \rightarrow y = \frac{x+14}{x-2} = f^{-1}(x) \rightarrow f^{-1}(y) = f(x) \rightarrow \frac{x+14}{x-2} = \frac{2x+14}{x-1} \rightarrow x^2 + 10x - 14 = 2x^2 + 14x - 3x^2$$

$$\rightarrow x^2 - 3x - 14 = 0 \rightarrow \text{معادله درجه دوم را حل کن} \rightarrow \frac{-b}{a} = \frac{3}{1} = 3 \quad \checkmark$$

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قدیمه بخودار نسبت به $y = u$ همان طرفین تابع است

برای طرفین کردن و ساده کردن:

$$y = u + f[u] \quad \text{از طرفین } f^{-1} \rightarrow [y] = [u] + f[u] = \delta[u] - [u] = \frac{[y]}{\delta}$$

$$y = u + f\left(\frac{[y]}{\delta}\right) \rightarrow u = y - f\left(\frac{[y]}{\delta}\right) \quad \text{طرفین} \rightarrow y = u - f\left(\frac{[y]}{\delta}\right)$$

$$(f-g)(u) = u + f[u] - u + f\left(\frac{[y]}{\delta}\right) = f, \delta[u]$$

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