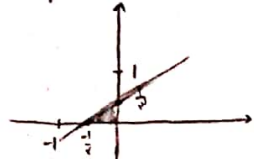


$$2y = 3x - 1 \rightarrow y = \frac{3x-1}{2} \Rightarrow f(n) = \frac{3n-1}{2} \Rightarrow y = \frac{3x-1}{2} \rightarrow 3x-1 = 2y \rightarrow 3x = 2y+1$$

$$\rightarrow x = \frac{2y+1}{3} \rightarrow y = \frac{3x-1}{2} \rightarrow f^{-1}(n) = \frac{3}{2}x + \frac{1}{2} \rightarrow \begin{cases} x=0, y=\frac{1}{2} \\ x=-\frac{1}{3}, y=0 \end{cases}$$


$$\rightarrow S_{\text{مثلث}} = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12}$$

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الف) $y = x^2 - 2x + 3, x \in [1, +\infty) \rightarrow \Delta < 0, \text{ent} \left| \frac{-b}{2a} \right| = \frac{1}{1} = 1$

$\rightarrow$ طبق شکل تابع در این بازه یک به یک است $\rightarrow y = x^2 - 2x + 3 = x^2 - 2x + 1 + 2 = (x-1)^2 + 2 \rightarrow y - 2 = (x-1)^2$

$\rightarrow y - 2 = (x-1)^2 \rightarrow \sqrt{y-2} = |x-1| \xrightarrow{x \geq 1} \sqrt{y-2} = x-1 \rightarrow x = \sqrt{y-2} + 1 \rightarrow f^{-1}(n) = \sqrt{n-2} + 1, D_{f^{-1}} = [2, +\infty)$

ب) $y = x^2 - 2x + 3, x \in [3, +\infty) \rightarrow \Delta < 0, \text{ent} \left| \frac{-b}{2a} \right| = \frac{1}{1} = 1$

$\rightarrow$ طبق شکل تابع در این بازه یک به یک است $\rightarrow y = x^2 - 2x + 3 = (x-1)^2 + 2 \rightarrow y - 2 = (x-1)^2 \rightarrow \sqrt{y-2} = |x-1| \xrightarrow{x \geq 3} \sqrt{y-2} = x-1 \rightarrow x = \sqrt{y-2} + 1 \rightarrow f^{-1}(n) = \sqrt{n-2} + 1, R_f = D_{f^{-1}} = [4, +\infty)$

$$f(x) = 2x - \sqrt{14 - 4x(f-x)} = 2x - \sqrt{f(f-x)(f-x)} = 2x - \sqrt{f^2 - x(f-x)} = 2x - \sqrt{f^2 - fx + x^2}$$

$$= 2x - \sqrt{(x-2)^2} = 2x - 2|x-2| \xrightarrow{x \geq 2} 2x - 2(x-2) = 2x - 2x + 4 = 4 \rightarrow y = 4 \rightarrow$$

در این بازه یک به یک نیست

$$\xrightarrow{x \leq 2} 2x - 2(-x+2) = 2x + 2x - 4 = 4x - 4 \rightarrow y = 4x - 4 \rightarrow 4x = y + 4 \rightarrow x = \frac{y+4}{4} \rightarrow y = \frac{4x-4}{1} = 4x-4$$

در این بازه یک به یک است

$$\rightarrow f(x) = 4x - 4, x \in (-\infty, 2] \rightarrow D_f = (-\infty, 2] \rightarrow R_f = (-\infty, 4]$$

$$\Rightarrow y = 4x - 4 \rightarrow 4x = y + 4 \rightarrow x = \frac{y+4}{4} \rightarrow y = \frac{4x-4}{1} = 4x-4$$

$$\rightarrow f^{-1}(n) = \frac{n+4}{4}, D_{f^{-1}} = R_f = (-\infty, 4] \rightarrow S = \frac{4 \times 1}{4} = 1$$


۳

$$y = \frac{x}{\sqrt{x^2-5}} \rightarrow \text{if } x = f(n) \text{ و } \sqrt{x^2-5} = g(n) \rightarrow y = h(n) = \frac{f(n)}{g(n)}$$

این تابع یک به یک است $\rightarrow$ الیاصوری $\rightarrow g = \frac{x}{\sqrt{x^2-5}}$

$$y = \frac{x}{\sqrt{x^2-5}} \rightarrow y^2 = \frac{x^2}{x^2-5} \rightarrow y^2(x^2-5) = x^2 \rightarrow y^2x^2 - 5y^2 = x^2 \rightarrow y^2x^2 - x^2 = 5y^2 \rightarrow x^2(y^2-1) = 5y^2$$

$$\rightarrow x^2 = \frac{5y^2}{y^2-1} \rightarrow x = \frac{\sqrt{5}y}{\sqrt{y^2-1}} \rightarrow y = \frac{x\sqrt{5}}{\sqrt{x^2-1}} \rightarrow f^{-1}(n) = \frac{n\sqrt{5}}{\sqrt{n^2-1}}$$

۴

اگر $x=2$ باشد ضابطه در تابع باید برابر باشد تا یک به یک شود تا ضابطه در اول که چتر شود.

$$\left. \begin{aligned} x=2 &\rightarrow f-1+k \\ x=2 &\rightarrow -f+12+3k \end{aligned} \right\} \rightarrow f-1+k = -f+12+3k \rightarrow k-f = 3k+11 \rightarrow 2k = -12 \rightarrow k = -6$$

$$-f+12+3k < f-1+k \rightarrow 1+3k < k-f \rightarrow 2k < -12 \rightarrow k < -6$$

$$\rightarrow k < -6 \rightarrow k \in (-\infty, -6]$$

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$$y = 3x + |x| \quad \begin{cases} x \geq 0 \rightarrow 3x + x = 4x \rightarrow y = 4x \rightarrow x = \frac{y}{4} \rightarrow f^{-1}(y) = \frac{y}{4}, & x \geq 0 \\ x < 0 \rightarrow 3x - x = 2x \rightarrow y = 2x \rightarrow x = \frac{y}{2} \rightarrow f^{-1}(y) = \frac{y}{2}, & x < 0 \end{cases}$$

$$\rightarrow f^{-1} \circ g(y) = a x + b |x| \quad \begin{cases} x \geq 0 \rightarrow a x + b x = \frac{x}{4} \rightarrow x(a+b) = \frac{x}{4} \rightarrow a+b = \frac{1}{4} \\ x < 0 \rightarrow a x - b x = \frac{x}{2} \rightarrow x(a-b) = \frac{x}{2} \rightarrow a-b = \frac{1}{2} \end{cases} \rightarrow \begin{cases} a+b = \frac{1}{4} \\ a-b = \frac{1}{2} \end{cases} \rightarrow \begin{cases} a = \frac{3}{8} \\ b = -\frac{1}{8} \end{cases}$$

$$\Rightarrow ab = \frac{3}{8} \times -\frac{1}{8} = \boxed{\frac{-3}{64}}$$

6

$$y = \frac{3x+f}{x+m} \xrightarrow{y=0} 0 = \frac{3x+f}{x+m} \rightarrow -10 = \frac{4+f}{1+m} \rightarrow -10m = 4+f \rightarrow 10m = -4-f \rightarrow m = -\frac{4+f}{10} \rightarrow f(x) = \frac{3x+f}{x-\frac{4+f}{10}}$$

$$\rightarrow f^{-1}(y) : y = \frac{3x+f}{x-\frac{4+f}{10}} \xrightarrow{ax+c=0} f^{-1}(y) = f(x) = \frac{3x+f}{x-\frac{4+f}{10}} \rightarrow g = \frac{f \circ f^{-1}(y)}{x} + \frac{f^{-1}(y)}{f(x)}$$

$$\rightarrow y = x + f(x) \rightarrow x + f(x) = x \rightarrow f(x) = 0 \rightarrow \frac{3x+f}{x-\frac{4+f}{10}} = 0 \rightarrow 3x+f=0$$

$$\rightarrow x = -\frac{f}{3} \rightarrow \text{لذلك } x = -\frac{f}{3} \text{ هو الحل}$$

7

$$f(3-2x) = y - g\left(\frac{x}{y}\right) \rightarrow f(3-2x) = t \xrightarrow{\text{نضع } t = f(3-2x)} f \circ f^{-1}(t) = f^{-1}(t) \rightarrow f^{-1}(t) = 3-2x$$

$$t = y - g\left(\frac{x}{y}\right) \rightarrow g\left(\frac{x}{y}\right) = y - t \xrightarrow{\text{نضع } t = y - g\left(\frac{x}{y}\right)} g \circ g^{-1}\left(\frac{x}{y}\right) = g^{-1}(y-t) \rightarrow g^{-1}(y-t) = \frac{x}{y}$$

$$\text{if } t = -y \rightarrow f^{-1}(-y) = 3 - 2(-y) = 5, \quad g^{-1}(y - (-y)) = g^{-1}(2y) = \frac{y}{2} = 1$$

$$\rightarrow 1 \cdot g^{-1}(2) + f^{-1}(-2) = ? = f(2) + 5 = \boxed{15}$$

8

$$y = \frac{2x-13}{1-x} \rightarrow -y = \frac{2(-x)-13}{1-(-x)} = \frac{-2x-13}{1+x} \leftarrow \text{نضع } -y = \frac{-2x-13}{1+x}$$

$$\rightarrow -y = \frac{-2x-13}{1+x} \rightarrow y = \frac{2x+13}{x+1} \xrightarrow{\text{نضع } y = \frac{2x+13}{x+1}} y = \frac{2(x-1)+13}{(x-1)+1} = \frac{2x-10+13}{x-1+1} = \frac{2x+3}{x-1}$$

$$\xrightarrow{\text{نضع } y = \frac{2x+3}{x-1}} \frac{2x+3}{x-1} - y = \frac{2x+3-y(x-1)}{x-1} = \frac{2x+3-yx+y}{x-1} = \frac{y+14}{x-1} \rightarrow f(x) = y = \frac{2x+14}{x-1} \rightarrow x = -\frac{y-14}{y-2}$$

$$\rightarrow x = \frac{y+14}{y-2} \rightarrow y = \frac{2x+14}{x-1} = f^{-1}(x) \rightarrow f^{-1}(y) = f(x) \rightarrow \frac{y+14}{y-2} = \frac{2x+14}{x-1} \rightarrow x^2 + 10x - 14 = 2x^2 + 14x - 2y^2$$

$$\rightarrow x^2 - 3x - 14 = 0 \rightarrow \text{لذلك } x = \frac{-b}{a} = \frac{3}{1} = \boxed{3}$$

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