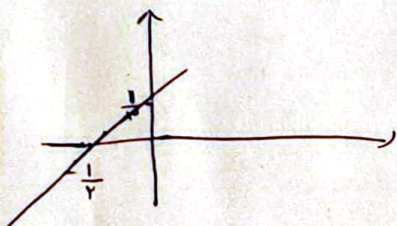


$f = 2y = 3x - 1 \Rightarrow f^{-1}(x) = \frac{2y+1}{3} = x \Rightarrow f^{-1}(x) = \frac{2x+1}{3}$



$S = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12} \checkmark$

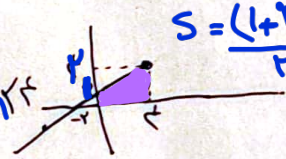
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الف) $x^2 - 2x + 2$ $x_S = 1$ $x \in (1, +\infty)$ $y = (x-1)^2 + 2 \Rightarrow \sqrt{y-2} = |x-1|$
 $\sqrt{y-2} + 1 = x \Rightarrow f^{-1}(x) = \sqrt{x-2} + 1$
 $D_{f^{-1}} = (2, +\infty) \checkmark$

ب) $x^2 - 2x + 2$ $x_S = 1$ $x \in (-\infty, 1)$ $y = (x-1)^2 + 2 \Rightarrow \sqrt{y-2} = |x-1|$
 $\sqrt{y-2} + 1 = x \Rightarrow f^{-1}(x) = \sqrt{x-2} + 1 = y$ $D_{f^{-1}} = (2, +\infty) \checkmark$

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$f(x) = 2x - \sqrt{4x^2 - 12x + 12} = 2x - 2\sqrt{x^2 - 3x + 3} = 2x - 2|x-2|$
 $f(x) = \begin{cases} 4x - 4 & x \geq 2 \\ 4x - 4 & x < 2 \end{cases}$ $\Rightarrow f^{-1}(x) = \frac{x+4}{4}$ $x \geq 2 \Rightarrow f^{-1}(x) = \frac{x+4}{4}$ $x < 2 \Rightarrow f^{-1}(x) = \frac{x+4}{4}$ $S = \frac{(4+2) \times 4}{2} = 12$

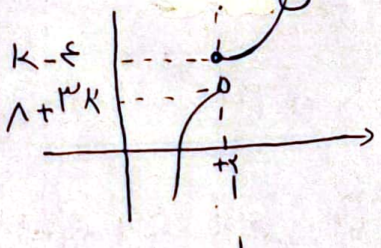


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$y^2 = \frac{x^2}{x^2 - 5} \Rightarrow (y^2 - 1)x^2 - 5y^2 = 0 \Rightarrow x^2 = \frac{5y^2}{y^2 - 1} \Rightarrow |x| = \sqrt{\frac{5y^2}{y^2 - 1}}$
 $f(x) = f(x_1) = \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}} \Rightarrow \frac{x_1^2}{x_1^2 - 5} = \frac{x_2^2}{x_2^2 - 5} \Rightarrow x_1^2(x_2^2 - 5) = x_2^2(x_1^2 - 5) \Rightarrow x_1^2 x_2^2 - 5x_1^2 = x_2^2 x_1^2 - 5x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$
 $f^{-1}(x) = \frac{\sqrt{5}y}{\sqrt{y^2 - 1}} = f^{-1}(x) = \frac{\sqrt{5}x}{\sqrt{x^2 - 1}} \checkmark$

۲

$k - \epsilon \geq 1 + 3k \Rightarrow k \leq -\frac{1}{2} \checkmark$



۲

$$f(n) \begin{cases} \leq n & n \geq 0 \\ \geq n & n < 0 \end{cases}$$

$$\Rightarrow f^{-1}(n) \begin{cases} \frac{n}{2} & n \geq 0 \\ \frac{n}{4} & n < 0 \end{cases}$$

$$n > 0 \quad (a+b) \times$$

$$n < 0 \quad (a-b) \times$$

$$a+b = \frac{1}{2}$$

$$a-b = \frac{1}{4}$$

$$\Rightarrow a = \frac{3}{4}$$

$$\Rightarrow b = -\frac{1}{4}$$

$$\Rightarrow ab = -\frac{3}{16} \checkmark$$

(2)

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$$f = \frac{3x+5}{x+m} \Rightarrow x=2 \quad y=-1 \Rightarrow \frac{y+5}{x+m} = -1 \Rightarrow m = -3$$

دقت!

(1)

$$\underbrace{f \circ f^{-1}(n)}_n + \underbrace{f^{-1}(n)}_{\frac{3x+5}{x-3}} = n \Rightarrow \frac{3x+5}{x-3} = 0 \Rightarrow x = -\frac{5}{3}$$

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$$f(3-2n) = a \quad g\left(\frac{n}{2}\right) = 2-a$$

$$f^{-1}(a) = 3-2n \quad g^{-1}(2-a) = \frac{n}{2} \Rightarrow 2g^{-1}(2-a) = 3-f^{-1}(a)$$

$$2g^{-1}(2-a) + f^{-1}(a) = 3 \xrightarrow{a=-2} 2g^{-1}(4) + f^{-1}(-2) = 3 \checkmark$$

$$a = -2$$

(2)

8

$$\text{تغییر } n \rightarrow -n$$

$$y \rightarrow -y \Rightarrow -y = \frac{-\omega n + 1}{1+n} \Rightarrow y = \frac{\omega n + 1}{1+n}$$

$$\frac{\omega(n-2)+1}{1+(n-2)} = \frac{\omega n + 1}{n-1} - 3 = \frac{2n+7}{n-1} = f(n) \quad f^{-1}(n) = \frac{n+2}{n-3}$$

$$\frac{2n+7}{n-1} = \frac{n+2}{n-3} \Rightarrow n^2 + \omega n - 7 = 1n^2 + 2n - 12 \Rightarrow n^2 - 3n - 5 = 0 \quad S = 3 \checkmark$$

(2)

9

$$y = x + 5 \cos \Rightarrow x = y + 5 \cos \Rightarrow y = x - 5 \cos$$

$$[n] = [y + 5 \cos] = 5 \cos \Rightarrow \cos = \frac{[n]}{5}$$

$$g(n) = f^{-1}(n) = y = x - \frac{5}{\omega} \cos \quad f(n) - g(n) = x + 5 \cos - x + \frac{5}{\omega} \cos = \frac{5\omega + 5}{\omega} \cos$$

(2)

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