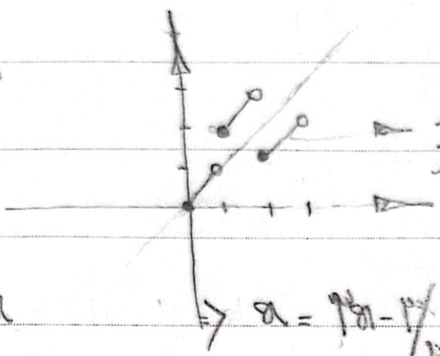
۱۵٫۷۵

۱) الف) $f(x) = x$ نقاط مشترک $f^{-1}(x)$ و

$f(x) = x + [x]$



$x + [x] = x \Rightarrow [x] = 0$ ۱٫۵

$\Rightarrow 0 \leq x < 1 \Rightarrow$ بی‌نهایت $[0, 1)$

ب) $f(f^{-1}(x)) = x \Rightarrow x = 2x - \frac{3}{2}$ $\quad x = \frac{3}{2}$

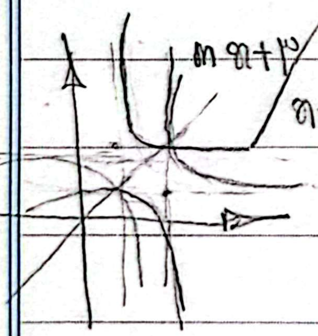
برد تابع f از هر عدد زوج تا عدد فرد بعدی است یعنی $\frac{3}{2} = x$ غلط می‌باشد و جواب تهی است!

$$\frac{a}{۴} = \frac{-b}{۴} \Rightarrow a = -b \qquad y = \frac{ax+۳}{۴x-a} \qquad f(f(x)) = \frac{a\left(\frac{ax+۳}{۴x-a}\right)+۳}{}$$ (۲

$$\Rightarrow \frac{a^۲x+\cancel{۳a}+۱۲x-\cancel{۳a}}{\cancel{۴ax}+۱۲-\cancel{۴ax}+a^۲} = \frac{x(\cancel{a^۲+۱۲})}{\cancel{a^۲+۱۲}} \Rightarrow f(f(x)) = x \qquad \frac{۴\left(\frac{ax+۳}{۴x-a}\right)-a}{} \quad = \boxed{۵-\sqrt{۹}}$$ ۲

$$f(x) = \frac{mx+۳}{x+m+۳} = y \qquad f^{-1}(x) = \frac{-(m+۳)x+۳}{x-m} = \frac{-mx+۳x+۳}{x-m}$$ ۲ (۳

$$mx+۳(x-m) = ۳x+۳-mx(x+m+۳)$$



$$\frac{mx+۳}{x+m+۳} = x$$

۳) اگر تابع f(x) و وارون آن همدیگر را قطع کنند نقاط تقاطع روی y=x است

$$\Rightarrow x^۲ + x\cancel{m} + ۳x = \cancel{m}x + ۳ \Rightarrow x^۲ + ۳x - ۳ = ۰$$

$$\frac{-b \pm \sqrt{b^۲-۴ac}}{۲a} = \frac{-۳ \pm \sqrt{۹+۱۲}}{۲} \Longrightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-b/a}{c/a} = \frac{-۳}{-۳} = ۱$$

۴- ۱٫۷۵

$f(x) = |2x+5| - |5x-2|$

	$-\frac{5}{2}$	$\frac{2}{5}$	
	$-2x-5+5x-2 = 3x-7$	$2x+5+5x-2 = 7x+3$	$2x+5-5x+2 = -3x+7$

$x = \frac{2}{5} \Rightarrow \frac{4}{5} + \frac{25}{5} = \frac{29}{5}$

$x = -\frac{5}{2} \Rightarrow -\frac{15}{2} - \frac{4}{2} = -\frac{19}{2}$

$f^{-1}(x) = -3y+7 = x \Rightarrow f^{-1}(x) = \frac{x-7}{-3}$ ✓

$x \in \cancel{\left[-\frac{19}{2}, \frac{29}{5}\right]}$

$x \in \left(-\infty, \frac{29}{5}\right]$

۵- ۲

$g(x) = 2f(3x) - 1 \qquad y = 2f(3x) - 1 \qquad \frac{y+1}{2} = f(3x)$

$f^{-1}\left(\frac{g(x)+1}{2}\right) = 3x \qquad x = \frac{1}{3} f^{-1}\left(\frac{g(x)+1}{2}\right) \Rightarrow g^{-1}(y) = \frac{1}{3} f^{-1}\left(\frac{y+1}{2}\right)$

$g^{-1}(x) = \frac{1}{3} f^{-1}\left(\frac{x+1}{2}\right) \Rightarrow a = \frac{1}{3}$ ✓ $, b = 1$ ✓ $, c = \frac{1}{2}$ ✓ $, d = 0$ ✓ $\Rightarrow \frac{ac+d}{2b} = \frac{\frac{1}{6}}{2} = \frac{1}{12}$ ✓

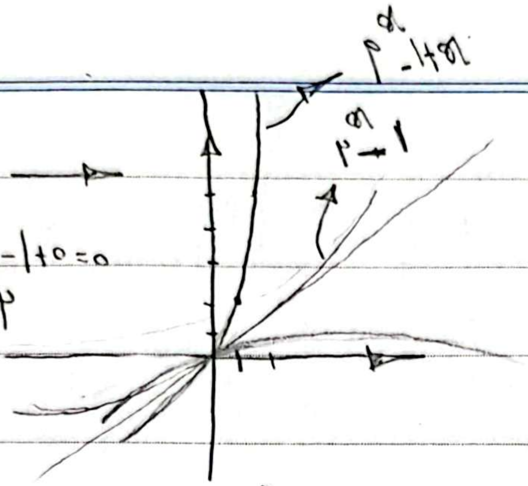
۶- ۲

$f(x) = x + 2\sqrt{x} \qquad x \geq 0 \qquad \sqrt{x} = t \Rightarrow y = t^2 + 2t \qquad t^2 + 2t - y = 0$

$y^2 = x^2 + 4x + 4x\sqrt{x} \qquad t = \frac{-2 \pm \sqrt{4+4y}}{2} \Rightarrow t = -1 + \sqrt{1+y}$

$\Rightarrow x = t^2 = \left(-1 + \sqrt{1+y}\right)^2 = y + 1 + 1 - 2\sqrt{1+y} = y + 2 - 2\sqrt{1+y} \overset{t \geq 0}{\Rightarrow} f^{-1}(x) = x + 2 - 2\sqrt{1+x}$ ✓

✓ $x \in [0, +\infty)$



۷_ $y = 2^x - 1 + x \longrightarrow$ ⇒ یک نقطه $x=0$ ✓

$x = 0 \to 1 - 1 + 0 = 0$

$x = 1 \to 2$

$x = 2 \to 5$

(۲)

۸_ (۰)

$f^{-1}(x) \geq x \to f(x) \leq x$

$x^3 + 4x \leq x \to x(x^2+3) \leq 0$

$\to x \leq 0$ شامل یک عدد صحیح نامثبت $\{0\}$

$f(x) = x^3 + 4x \Rightarrow f(x^3 + 4x) = x$

$\cancel{x - x^3 - 4x} \geq 0 \quad -x^3 - 3x \geq 0 \quad -(x)(x^2+3) \geq 0$

	$-\sqrt{3}$	0	$+\sqrt{3}$	
x	$-$	$-$	$+$	$+$
x^2+3	$+$	$-$	$-$	$+$
	$-$	$+$	$-$	$+$

$\Rightarrow [0, +\sqrt{3}] \cup (-\infty, -\sqrt{3}]$

$\Rightarrow \cancel{\{0\}}$

۹_ (۲)

$y = -x + \sqrt{x+4} \quad x \geq -3 \quad (y+x)^2 = x + 4 \Rightarrow$

$x^2 + (2y-1)x + (y^2 - 4) = 0 \quad x = \dfrac{1 - 2y \pm \sqrt{(2y-1)^2 - 4(y^2-4)}}{2}$

$= \dfrac{1 - 2y \pm \sqrt{\cancel{4y^2} + 1 - 4y \cancel{- 4y^2} + 16}}{2} = \dfrac{1 - 2y \pm \sqrt{17 - 4y}}{2} \xrightarrow[x \geq -3]{}$

$+3 + \sqrt{1} = 4 \qquad f^{-1}(x) = \dfrac{1 - 2x + \sqrt{17 - 4x}}{2} \to y \leq 17/4$

$g(x) = \dfrac{1 - 2(x+4) + \sqrt{17 - 4x - 16}}{2} \to 1 - 4x \geq 0 \quad x \leq 1/4$

$\dfrac{-2x - 7 + \sqrt{1 - 4x}}{2} = 2x - 4 \qquad 4x + 1 = \sqrt{1 - 4x} \Rightarrow 4x + 1 \geq 0 \Rightarrow x \geq -1/4$

$-1/4 \leq x \leq 1/4 \qquad \xrightarrow{\text{به توان ۲}} 16x^2 + 1 + 8x = 1 - 4x \Rightarrow 16x^2 + 12x = 0$

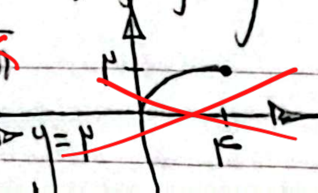
$\Rightarrow x(4x+3) = 0 \quad x = 0$ ✓ $x = -3/4$ ✗ $\Rightarrow g(0) = 0 - 3 = -3$ ✓ ۱ نمره

۱۰_ (۰/۵)

$\sqrt{x} + \sqrt{y} = 2 \Rightarrow \sqrt{y} = 2 - \sqrt{x} \quad y = (2 - \sqrt{x})^2 \quad 2 - \sqrt{x} \geq 0$

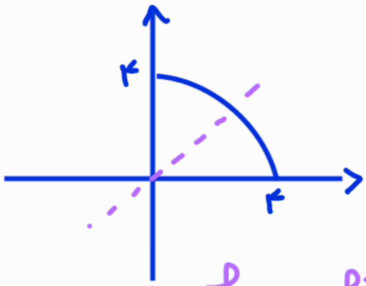
$0 \leq \sqrt{x} \leq 2 \Rightarrow 0 \leq x \leq 4$ ✓ $\Rightarrow f \circ f(x) = 2 - \sqrt{(2 - \sqrt{x})^2} = 2 - |2 - \sqrt{x}|$

$\Rightarrow f \circ f(x) = 2 - 2 + \sqrt{x} \cancel{- 4x}$

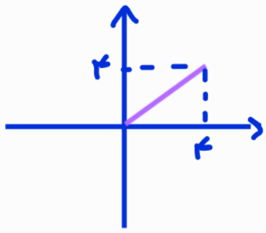


$x = 4 \to y = 2$

$\sqrt{y} = ۲ - \sqrt{x} \geq 0 \to 0 \leq x \leq ۴$



$f \circ f(x) = f \circ f^{-1}(x) = x$



$f(x) = f^{-1}(x)$

$D_f = R_f$