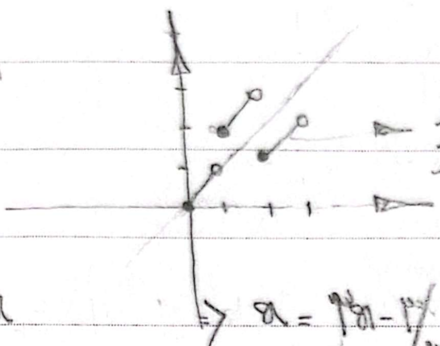
[illegible]

۱) الف) $f(x) = x$ نقاط مشترک $f^{-1}(x)$

$f(x) = x + [x]$



$x + [x] = x \Rightarrow [x] = 0$

$\Rightarrow 0 \leq x < 1 \Rightarrow$ جواب $[0, 1)$

ب)

$f(f^{-1}(x)) = x$

$\Rightarrow x = 9x - \frac{12}{2}$ $\quad x = \frac{12}{10}$

$۱+۵$

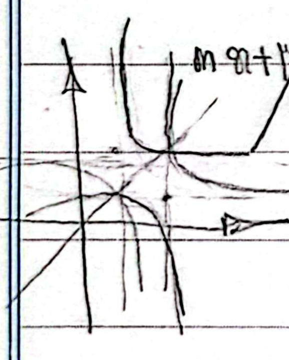
$\frac{a}{۴} = \frac{-b}{۴} \Rightarrow a = -b \qquad y = \frac{ax+۳}{۴x-a} \qquad f(f(x)) = \frac{a\left(\frac{ax+۳}{۴x-a}\right)+۳}{۴\left(\frac{ax+۳}{۴x-a}\right)-a}$ (۲

$\Rightarrow \frac{a^۲x+۳a+۱۲x-۳a}{۴ax+۱۲-۴ax+a^۲} = \frac{x(a^۲+۱۲)}{a^۲+۱۲} \Rightarrow f(f(x)) = x \quad = ۵-\sqrt{۹}$

$f(x) = \frac{mx+۳}{x+m+۳} = y \qquad f^{-1}(x) = \frac{-(m+۳)x+۳}{x-m} = \frac{-mx+۳x+۳}{x-m}$ (۳

$mx+۳(x-m) = ۳x+۳-mx(x+m+۳)$

۳) اگر تابع f(x) با وارون خود را قطع کند باید نقاط تقاطع روی y=x باشند



$\frac{mx+۳}{x+m+۳} = x$

$\Rightarrow x^۲ + x\cancel{m} + ۳x = \cancel{m}x + ۳ \Rightarrow x^۲ + ۳x - ۳ = ۰$

$\frac{-b\pm\sqrt{b^۲-۴ac}}{۲a} = \frac{-۳\pm\sqrt{۹+۱۲}}{۲} \Longrightarrow \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-b/a}{c/a} = \frac{-۳}{-۳} = ۱$

4-

$$f(x) = |2x+5| - |5x-2| \qquad -\tfrac{5}{2} \qquad \tfrac{2}{5}$$

$$-2x-5+5x-2 = 3x-7 \qquad 2x+5+5x-2 = 7x+3 \qquad 2x+5-5x+2 = -3x+7$$

$$x = \tfrac{2}{5} \Rightarrow \tfrac{14}{5} + \tfrac{15}{5} = \tfrac{29}{5}$$

$$x = -\tfrac{5}{2} \Rightarrow -\tfrac{15}{2} - \tfrac{14}{2} = -\tfrac{29}{2}$$

$$f^{-1}(x) = -3y+7 = x \Rightarrow f^{-1}(x) = \frac{x-7}{-3} \qquad x \in \left[-\tfrac{29}{2}, \tfrac{29}{5}\right]$$

5-

$$g(x) = 2f(3x) - 1 \qquad y = 2f(3x) - 1 \qquad \frac{y+1}{2} = f(3x)$$

$$f^{-1}\left(\frac{g(x)+1}{2}\right) = 3x \qquad x = \frac{1}{3} f^{-1}\left(\frac{g(x)+1}{2}\right) \Rightarrow g^{-1}(y) = \frac{1}{3} f^{-1}\left(\frac{y+1}{2}\right)$$

$$g^{-1}(x) = \frac{1}{3} f^{-1}\left(\frac{x+1}{2}\right) \Rightarrow a = \frac{1}{2},\ b = 1,\ c = 3,\ d = 0 \Rightarrow \frac{ac+d}{3b} = \frac{3/2}{3} = \frac{1}{2}$$

6-

$$f(x) = x + 2\sqrt{x} \qquad x \geq 0 \qquad \sqrt{x} = t \Rightarrow y = t^2 + 2t \qquad t^2 + 2t - y = 0$$

$$y^2 = x^2 + 4x + 4x\sqrt{x} \qquad t = \frac{-2 \pm \sqrt{4+4y}}{2} \Rightarrow t = -1 + \sqrt{1+y}$$

$$\Rightarrow x = t^2 = \left(-1 + \sqrt{1+y}\right)^2 = y + 1 + 1 - 2\sqrt{1+y} = y + 2 - 2\sqrt{1+y} \overset{t \geq 0}{\Rightarrow} f^{-1}(x) = x + 2 - 2\sqrt{1+x}$$

$$x \in [0, +\infty)$$

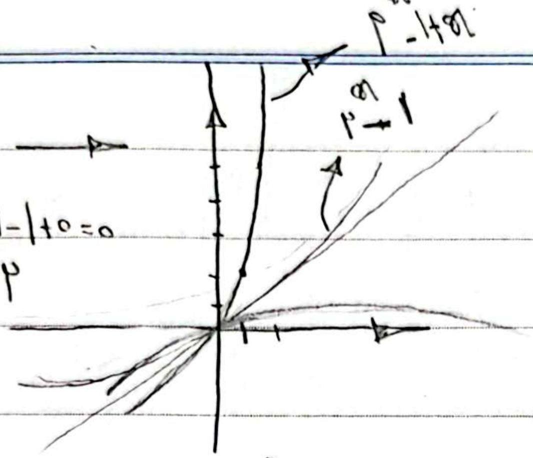
7_ $x=0$ ⇒ [illegible]

$y = 2^x - 1 + x$

$x = 0 \to 1 - 1 + 0 = 0$

$x = 1 \to 2$

$x = 2 \to 5$



8_ $f(x) = x^3 + 4x \Rightarrow f(x^3 + 4x) = x$

$x - x^3 + 4x \geq 0 \quad -x^3 + 3x \geq 0 \quad -(x)(x^2 + 3) \geq 0$

	$-\sqrt{3}$	0	$+\sqrt{3}$	
x	$-$	$-$	$+$	$+$
x^2+3	$+$	$-$	$-$	$+$
	$-$	$+$	$-$	$+$

$\Rightarrow [0, +\sqrt{3}] \cup (-\infty, -\sqrt{3}]$

$\Rightarrow \{0, 1\}$ → $x<2$

9_ $y = -x + \sqrt{x + 4} \quad x \geq -2 \quad (y + x)^2 = x + 4 \Rightarrow$

$x^2 + (2y - 1)x + (y^2 - 4) = 0 \quad x = \dfrac{1 - 2y \pm \sqrt{(2y-1)^2 - 4(y^2 - 4)}}{2}$

$= \dfrac{1 - 2y \pm \sqrt{\cancel{4y^2} + 1 - 4y \cancel{- 4y^2} + 16}}{2} = \dfrac{1 - 2y \pm \sqrt{17 - 4y}}{2} \xrightarrow{x \geq -2}$

$+3 + \sqrt{1} = 4 \qquad f^{-1}(x) = \dfrac{1 - 2x + \sqrt{17 - 4x}}{2} \to y \leq 17/4$

$g(x) = \dfrac{1 - 2(x + 4) + \sqrt{17 - 4x - 16}}{2} \to 1 - 4x \geq 0 \quad x \leq 1/4$

$\dfrac{-2x - 7 + \sqrt{1 - 4x}}{2} = 2x - 4 \qquad 4x + 1 = \sqrt{1 - 4x} \Rightarrow 4x + 1 \geq 0 \Rightarrow x \geq -1/4$

$-1/4 \leq x \leq 1/4 \qquad \xrightarrow{\text{توان } 2} 16x^2 + 1 + 8x = 1 - 4x \Rightarrow 16x^2 + 12x = 0$

$\Rightarrow x(4x + 3) = 0 \quad x = 0 \ ✓ \quad x = -3/4 \ ✗ \Rightarrow g(0) = 0 - 3 = -3$ [illegible]

10_ $\sqrt{x} + \sqrt{y} = 2 \Rightarrow \sqrt{y} = 2 - \sqrt{x} \quad y = (2 - \sqrt{x})^2 \quad 2 - \sqrt{x} \geq 0$

$0 \leq \sqrt{x} \leq 2 \Rightarrow 0 \leq x \leq 4 \qquad \Rightarrow f \circ f(x) = 2 - \sqrt{(2 - \sqrt{x})^2} = 2 - |2 - \sqrt{x}|$

$\Rightarrow f \circ f(x) = 2 - 2 + \sqrt{x} = \sqrt{x}$

$x = 4 \to y = 2$