

$$f(x) = x + [x] \rightarrow y = x + [x] \rightarrow [y] = [x + [x]] \rightarrow [y] = 2[x]$$

$$y = x + [x] \rightarrow x = y + [y] \rightarrow x = y + 2[x] \rightarrow y = x - 2[x] \rightarrow f^{-1}(x) = x - 2[x]$$

$$الف) f(x) = f^{-1}(x) \rightarrow x + [x] = x - 2[x] \rightarrow 3[x] = 0 \rightarrow [x] = 0 \quad (0 \leq x < 1)$$

$$ب) f \circ f^{-1}(x) = 2x - \frac{3}{4} \rightarrow x = 2x - \frac{3}{4} \rightarrow x = \frac{3}{4}$$

$$f\left(\frac{3}{4}\right) = \frac{3}{4} + [1.5] = 2.5 \quad \times$$

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$$y = \frac{ax+b}{cx+d} \rightarrow \frac{a}{c} = -\frac{b}{d} \rightarrow \frac{a}{c} = -\frac{b}{d} \rightarrow a = -b$$

$$f \circ f(0 - \sqrt{4}) = f \circ f^{-1}(8 - \sqrt{4}) = (8 - \sqrt{4})$$

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$$f(x) = \frac{mx+n}{x+m+n} = y \rightarrow x = -\frac{(m+n)y-n}{y-m} \rightarrow f^{-1}(x) = \frac{mx+nx-n}{-x+m}$$

$$f(x) = f^{-1}(x) \rightarrow \frac{mx+n}{x+m+n} = \frac{mx+nx-n}{-x+m} \rightarrow (2m+n)x^2 + (4m+n)x - 4m-n = 0$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{S}{P} = \frac{-\frac{b}{a}}{\frac{c}{a}} = -\frac{b}{c} = \frac{-4m-n}{-4m-n} = 1$$

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$$f(x) = \left| \frac{x}{2} + 1 \right| - \left| \frac{x}{2} - 1 \right| \quad \begin{array}{c} -\frac{1}{2} \quad \frac{1}{2} \\ \frac{x}{2} - 1 \quad \frac{x}{2} + 1 \end{array}$$

$$y = -\frac{x}{2} + 1 \xrightarrow{y=x} x = -2y + 2$$

$$y = \frac{2-x}{2} : x \leq 2$$

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$$y = r f(rx) - 1 \rightarrow f(rx) = \frac{y+1}{r}$$

$$f^{-1}\left(\frac{y+1}{r}\right) = rx \rightarrow x = \frac{f^{-1}\left(\frac{y+1}{r}\right)}{r}$$

$$\left. \begin{array}{l} a = \frac{1}{r} \\ b = 1 \\ c = r \\ d = - \end{array} \right\} \frac{ac+d}{rb} = \frac{1}{r}$$

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$$f(x) = x + 2\sqrt{x} \quad Df = [0, +\infty) \quad Rf = [0, +\infty) \rightarrow Df^{-1} = [0, +\infty) \quad Rf^{-1} = [0, +\infty)$$

$$f(x) = x + 2\sqrt{x} + 1 - 1 \rightarrow y = (\sqrt{x} + 1)^2 - 1 \xrightarrow{\text{جایگزینی}} x = (\sqrt{y} + 1)^2 - 1$$

$$\sqrt{x+1} = \sqrt{y} + 1 \quad y = (\sqrt{x+1} - 1)^2 \rightarrow f^{-1}(y) = x = y - 2\sqrt{y+1} + 2 \quad x \geq 0$$

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$$y = x - 1 + 2^x$$

این تابع اکبراً معکوس است پس وارون خود را روی خط $y=x$ قطع می کند

$$y=x \rightarrow x = x - 1 + 2^x$$

$$2^x = 1$$

$$x=0$$

در این نقطه وارون خود را قطع می کند.

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$$y = \sqrt{f^{-1}(n) - n} \quad f^{-1}(n) - n \geq 0 \quad f^{-1}(n) \geq n \rightarrow f(f^{-1}(n)) \geq f(n)$$

$$f(n) \leq n \quad n^3 + \varepsilon n \leq n \rightarrow n^3 + 3n \leq 0 \rightarrow n(n^2 + 3) \leq 0$$

$$\frac{0}{- \quad +} \rightarrow D = (-\infty, 0] \leftarrow \text{شامل یک عدد صحیح نامقر}$$

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$$f(x) = -x + \sqrt{x+\varepsilon} \rightarrow y = -(x - \sqrt{x+\varepsilon} + \frac{14}{\varepsilon} - \frac{14}{\varepsilon}) \Rightarrow y = -(x + \varepsilon - \sqrt{x+\varepsilon} + \frac{1}{\varepsilon}) + \frac{14}{\varepsilon} = -(\sqrt{x+\varepsilon} - \frac{1}{\varepsilon})^2 + \frac{14}{\varepsilon}$$

$$-y + \frac{14}{\varepsilon} = (\sqrt{x+\varepsilon} - \frac{1}{\varepsilon})^2 \xrightarrow{n \geq -\frac{14}{\varepsilon}} \sqrt{\frac{14}{\varepsilon} - y} + \frac{1}{\varepsilon} = \sqrt{x+\varepsilon} \xrightarrow{n \geq -\frac{14}{\varepsilon}} (\sqrt{\frac{14}{\varepsilon} - y} + \frac{1}{\varepsilon})^2 - \varepsilon = x$$

$$y = (\sqrt{\frac{14}{\varepsilon} - n} + \frac{1}{\varepsilon})^2 - \varepsilon \xrightarrow{n \geq -\frac{14}{\varepsilon}} (\sqrt{\frac{1}{\varepsilon} + y} + \frac{1}{\varepsilon})^2 - \varepsilon = y = g(n) \rightarrow (\sqrt{\frac{1}{\varepsilon} - n} + \frac{1}{\varepsilon})^2 - \varepsilon = n - \frac{14}{\varepsilon}$$

$$\sqrt{\frac{1}{\varepsilon} - n} + \frac{1}{\varepsilon} = \sqrt{n-1} \leftarrow \text{فاصله نقطه بر صفر}$$

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$$\sqrt{x} + \sqrt{y} = 2 \xrightarrow{\text{جایگزینی معین}} \sqrt{y} + \sqrt{x} = 2$$

تابع وارون یک به یک

$$f \circ f^{-1}(n) = f \circ f^{-1}(n)$$

$$f \circ f^{-1}(n) = n \quad x \geq 0$$

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