

مرتبه اول - پاسخ به سؤالات تکمیلی شماره ۱۱

۱- $f(x) = x + [x]$

الف) $f(x) = x + [x] \xrightarrow{[]} [y] = [x + [x]] \rightarrow [x] = \frac{[y]}{2}$

$f^{-1} \rightarrow x + \frac{[y]}{2} = y \rightarrow y = x + \frac{[x]}{2} = f^{-1}(x)$

$\rightarrow x - \frac{[x]}{2} = x + \frac{[x]}{2} \rightarrow 2[x] = -[x] \Rightarrow 3[x] = 0 \rightarrow [x] = 0$

$x \in [0, 1) \rightarrow$ به کار می آید

ب) $f \circ f^{-1}(x) = x - \frac{3}{2}$

$f \circ f^{-1}(x) = x = x - \frac{3}{2} \rightarrow$

$x = \frac{3}{2} \rightarrow$ در این صورت

$x \in D = \{(2k, 2k+1) : k \in \mathbb{Z}\}$

جواب ندارد

$f \circ f^{-1}(2 - \sqrt{4}) = 0$

$y = \frac{ax + 3}{fx + b}$

۲-

مجاوبه ها: $\frac{a}{f}$

مجاوبه قائم $\rightarrow fx + b = 0 \rightarrow fx = -b \rightarrow x = -\frac{b}{f}$

$\rightarrow (\frac{a}{f}, \frac{a}{f}) = (-\frac{b}{f}, \frac{b}{f})$

$y = \frac{-bx + 3}{fx + b} \xrightarrow{b + (-b) = 0} f(x) = f^{-1}(x) \rightarrow f \circ f^{-1}(x) = x$

$f \circ f^{-1}(2 - \sqrt{4}) = 2 - \sqrt{4}$

PAYCO

DATE :

$$\frac{1}{\alpha} + \frac{1}{\beta}$$

β, α

$$f(x) = \frac{mx + r}{x + m + r} \quad - \mu$$

$$f(\alpha) = f^{-1}(\alpha)$$

$$f(\beta) = f^{-1}(\beta)$$

die $\beta, \alpha \leftarrow f^{-1} \circ f$ sind identisch

$$\frac{mx + r}{x + m + r} = x \rightarrow mx + r = x^2 + mx + rx \rightarrow x^2 + rx - rx - rx + r = 0 \rightarrow x^2 - 5rx + r = 0$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{5}{r} = \frac{-r}{-r} = 1$$

$$f^{-1} = g$$

$$f(x) = \sqrt{(rx + a)^2} - \sqrt{(ax - r)^2} \quad - K$$

$$f(x) = |rx + a| - |ax - r|$$

$$\begin{array}{ccc} -\frac{a}{r} & \frac{r}{a} & \\ \hline -rx - a & rx + a & rx + a - ax + r \\ rx - a & rx + a & rx + a - ax + r \\ rx - a & rx + a & rx + a - ax + r \\ rx - a & rx + a & rx + a - ax + r \end{array}$$

$$y = -rx + v$$

$$y - v = -rx$$

$$x = \frac{v - y}{r}$$

$$f^{-1}(x) = \frac{v - x}{r}$$

$$\frac{ac + d}{rb}$$

$$g(x) = rf(rx) - 1 \quad - \omega$$

$$g^{-1}(x) = a f^{-1}\left(\frac{x + b}{r}\right) + d$$

$$g(x) = rf(rx) - 1 \xrightarrow{g^{-1}} y = rf(rx) - 1 \rightarrow \frac{x + 1}{r} = f(rx)$$

$$\rightarrow f^{-1}\left(\frac{x + 1}{r}\right) = rx \rightarrow y = \frac{f^{-1}\left(\frac{x + 1}{r}\right)}{r}$$

$$b = 1$$

$$c = r$$

$$a = \frac{1}{r}$$

$$d = 0$$

$$\frac{ac + d}{rb} = \frac{\frac{1}{r} \cdot r}{r} = \frac{1}{r}$$

PAYCO

DATE:

$$D_{f^{-1}} = x \geq 0 \quad \text{بازگشایی} \quad f(x) = x + 2\sqrt{x} + 1 - 1 - 9$$

$$f(x) = (\sqrt{x} + 1)^2 - 1 \quad x \geq 0$$

$$y + 1 = (\sqrt{x} + 1)^2 \rightarrow \sqrt{y+1} = \sqrt{x} + 1 \quad y \geq 0$$

$$\sqrt{y+1} - 1 = \sqrt{x} \rightarrow x = y + 1 + 1 - 2\sqrt{y+1}$$

$$f^{-1}(y) = x = (\sqrt{y+1} - 1)^2 + 1 \leftarrow x = y - 2\sqrt{y+1} + 2$$

$$y = x - 2\sqrt{x+1} + 2$$

$$-V \quad y = x - 1 + 2^x \quad \text{دارد در این نقطه قطع}$$

هر تابع صعودی آید $\leftarrow$ جمع ۲ تابع صعودی آید $\leftarrow$ معادس در $y = x$

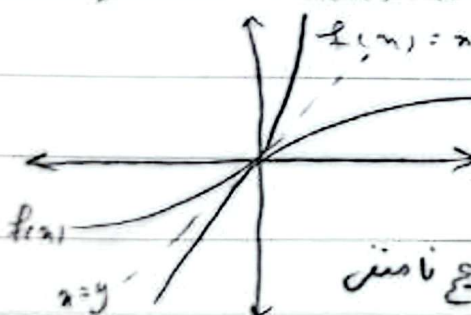
$$x = x - 1 + 2^x \rightarrow 1 = 2^x \rightarrow x = 0$$

فقط نقطه ۰ در ۰

$$y = \sqrt{f^{-1}(x) - x}$$

$$f(x) = x^3 + 4x - 1$$

$$f^{-1}(x) \geq x \rightarrow f^{-1}(x) = x \rightarrow x^3 + 4x = x$$



$$D = (-\infty, 0]$$

در x های منفی بالاتر $\leftarrow$ شامل ۱ عدد صحیح نامنظم

$$\frac{x^3 + 4x}{\text{منفی}} \rightarrow 3x^2 + 4 = 0 \rightarrow 3x^2 = -4 \rightarrow \text{جواب ندارد}$$

min و max ندارد

PAYCO

DATE:

$$f(x) = -x + \sqrt{x+f} \quad x \geq -r - 1$$

$$x+y = \sqrt{y+f} \rightarrow (x+y)^2 = y+f \rightarrow x^2 + 2xy + y^2 - y - f = 0$$

$$\rightarrow y^2 + (2x-1)y + (x^2-f) = 0 \rightarrow y = \frac{-(2x-1) \pm \sqrt{(2x-1)^2 - 4(x^2-f)}}{2}$$

$$f x^2 - \varepsilon x + 1 - f x^2 + 14 = 14 - \varepsilon x$$

$$y = \frac{-2x+1 \pm \sqrt{14-\varepsilon x}}{2} \xrightarrow{\text{choosing the positive root}} y = \frac{-2x+1 + \sqrt{14-\varepsilon x}}{2} \rightarrow \text{so, } f$$

$$g(x) = \frac{-1(x+\varepsilon) + 1 + \sqrt{14-\varepsilon(x+\varepsilon)}}{2} \rightarrow \frac{-x-\varepsilon+1+\sqrt{14-\varepsilon x}}{2} = x - \varepsilon \rightarrow x = 0$$

for $f(x)$

$$\sqrt{x} + \sqrt{y} = r - 1$$

$$\sqrt{x} + \sqrt{y} = r \xrightarrow{f(x)} \sqrt{y} + \sqrt{x} = r \rightarrow f^{-1}(x) = f(x)$$

$$D_f = [0, \varepsilon] \quad R_f = [0, \varepsilon]$$

$\sqrt{x}$ و $\sqrt{y}$ دالة متزايدة

$$f \circ f(x) = f^{-1} \circ f(x) = x \rightarrow D_{f^{-1}} = R_f = [0, \varepsilon]$$

