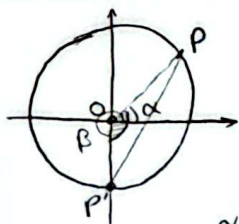


$$\frac{a \sin \frac{\pi}{4} + b \cos \frac{\pi}{4}}{a \sin \frac{\pi}{4} + b \cos \frac{\pi}{4}} = \tan \frac{\pi}{4}$$

$$\frac{a \sin(\frac{\pi}{4} - \frac{\pi}{4}) + b \cos(\frac{\pi}{4} + \frac{\pi}{4})}{a \sin(\frac{\pi}{4} - \frac{\pi}{4}) + b \cos(\frac{\pi}{4} - \frac{\pi}{4})} = \frac{-a \cos \frac{\pi}{4} - b \cos \frac{\pi}{4}}{-a \sin \frac{\pi}{4} + b \cos \frac{\pi}{4}} = \frac{-\frac{\sqrt{r}}{r}(a+b)}{-\frac{1}{r}(a-\sqrt{r}b)} = \frac{\sqrt{r}a + \sqrt{r}b}{a - \sqrt{r}b} = \frac{-1}{\sqrt{r}}$$

$$\Rightarrow \sqrt{r}a + \sqrt{r}b = \sqrt{r}b - a \Rightarrow (\sqrt{r} - \sqrt{r})b = -\sqrt{r}a \Rightarrow \frac{b}{a} = \frac{r}{\sqrt{r} - r}$$



$$\frac{\cos \alpha}{\sin \alpha} = \frac{1}{\sqrt{r}} \Rightarrow \alpha = \frac{\pi}{4}$$

$$(\frac{1}{\sqrt{r}})^2 + (y)^2 = 1 \Rightarrow y = \pm \sqrt{1 - \frac{1}{r}} \Rightarrow \pm \sqrt{\frac{r-1}{r}}$$

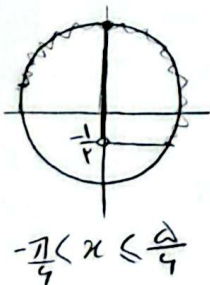
$$\alpha + \pi = \alpha + \frac{\pi}{4} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} = \beta$$

$$\Rightarrow \hat{O} = \frac{\pi}{4} + \frac{\pi}{4} = 180^\circ \Rightarrow PP' = \sqrt{OP^2 + OP'^2 - 2OP \cdot OP' \cos \hat{O}} \Rightarrow \sqrt{1 + 1 - 2 \times 1 \times 1 \times \cos 180^\circ} = \sqrt{2 + 2} = \sqrt{4} = 2$$

$$\Rightarrow \frac{\pi}{4} = \frac{r_{\min}}{?} \Rightarrow ? = \frac{\pi}{4} \times 40 = r_{\min} \Rightarrow 12.50$$

$$\theta = |d_1 d_2 M - r_0 h| \Rightarrow r_0 = |d_1 d_2 M - 90| \Rightarrow d_1 d_2 M - 90 = -r_0 \Rightarrow d_1 d_2 M + r_0 = 90 \Rightarrow M = \frac{90}{d_1 d_2} = 12.50$$

$$\hookrightarrow d_1 d_2 M - 90 = r_0 \Rightarrow d_1 d_2 M = 110 \Rightarrow M = \frac{110}{d_1 d_2} = 12.50$$



$$\Rightarrow -\frac{1}{r} < \sin x \leq 1 \Rightarrow -\frac{1}{r} < \frac{m-r}{a} \leq 1 \Rightarrow -\frac{a}{r} < m-r \leq a$$

$$\frac{1}{r} < m \leq 1$$

$$1 < m \leq 1$$

$$\frac{\sin(10^\circ) + k \cos(10^\circ)}{\cos(10^\circ) + r \sin(10^\circ)} = r \Rightarrow \frac{\sin(\frac{\pi}{18} + 10^\circ) + k \cos(\frac{\pi}{18} - 10^\circ)}{\cos(\frac{\pi}{18} - 10^\circ) + r \sin(\frac{\pi}{18} + 10^\circ)} = \frac{\cos 10^\circ + k \sin 10^\circ}{-\cos 10^\circ - r \sin 10^\circ}$$

$$\frac{1 - k \tan 10^\circ}{-1 - r \tan 10^\circ} = \frac{1 - 0.75k}{-1.07} = r \Rightarrow 1 - \frac{1}{2}k = -\frac{9}{r} \Rightarrow \frac{1}{2}k = 1 + \frac{9}{r} = \frac{11}{r} \Rightarrow k = \frac{22}{r}$$

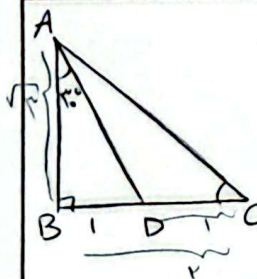
$$\frac{1 - \cos \alpha}{1 + \cos \alpha} = r \Rightarrow 1 - \cos \alpha = r + r \cos \alpha \Rightarrow r \cos \alpha = -1 \Rightarrow \cos \alpha = -\frac{1}{r}$$

$$\Rightarrow \sin \alpha = \sqrt{1 - \cos^2 \alpha} \Rightarrow \sqrt{1 - \frac{1}{r^2}} = \frac{\sqrt{r^2 - 1}}{r} \Rightarrow \tan \alpha = -\sqrt{r}, \cot \alpha = -\frac{1}{\sqrt{r}}$$

$$\frac{\tan(\frac{\pi}{4} - \alpha) - \cot(\frac{\pi}{4} - \alpha)}{\cot(\frac{\pi}{4} - \alpha) + \tan(\frac{\pi}{4} - \alpha)} = \frac{\cot \alpha + \cot \alpha}{\tan \alpha - r \tan \alpha} = \frac{r \frac{\cos \alpha}{\sin \alpha}}{-\frac{\sin \alpha}{\cos \alpha}} = -r \cot \alpha = -r \times \frac{1}{\sqrt{r}} = \boxed{-\frac{1}{\sqrt{r}}}$$

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \Rightarrow A = \frac{1}{2} \times \pi \times r_A = \frac{9}{10} \pi \times r_A \Rightarrow \frac{9}{10} \pi r_A = \frac{9}{10} \pi r_B \Rightarrow \frac{r_A}{r_B} = \frac{9}{9} = \frac{r}{r}$$

$$\frac{S_A}{S_B} = \frac{\pi r_A^2}{\pi r_B^2} = \left(\frac{r_A}{r_B}\right)^2 \Rightarrow \left(\frac{r}{r}\right)^2 = \boxed{\frac{9}{9}}$$



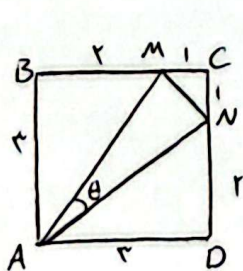
$$\tan 45^\circ = \frac{BD}{AB} \Rightarrow \frac{1}{\sqrt{r}} = \frac{1}{AB} \Rightarrow AB = \sqrt{r}$$

$$S_{ABC} = \frac{AB \times BC}{2} \Rightarrow r = \frac{\sqrt{r} \times BC}{2} \Rightarrow BC = r$$

$$\sin C = r \sin C \Rightarrow \sin C = r \times \frac{\sqrt{r}}{\sqrt{r}} \times \frac{1}{\sqrt{r}} = \boxed{\frac{r\sqrt{r}}{r}}$$

$$\sin C = \frac{\sqrt{r}}{\sqrt{r}}$$

$$\cos C = \frac{r}{\sqrt{r}}$$



$$AM = AN \Rightarrow \sqrt{r^2 + r^2} = \sqrt{1r^2}$$

$$(ABM, ADN \text{ congruent})$$

$$MN = \sqrt{r^2 + r^2} = \sqrt{r}$$

$$\Rightarrow MN^2 = AN^2 + AM^2 - 2AN \times AM \cos \theta \Rightarrow r = 1r^2 + 1r^2 - 2 \times \sqrt{r} \times \sqrt{r} \times \cos \theta$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \left(\frac{r}{r}\right)^2} = \boxed{\frac{1}{r}}$$

$$\cos \theta = \frac{r}{r} = \frac{1}{r}$$