(ریاضیات) ۱۹

①

$$\frac{a\sin\frac{4\pi}{3} + b\cos\frac{7\pi}{6}}{a\sin\frac{11\pi}{6} + b\cos\frac{\pi}{3}} = \tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$$

$\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$, $\cos(\pi+\frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$, $-\frac{1}{2}$, $\frac{1}{2}$

$$\frac{a(-\frac{\sqrt{3}}{2})+b(-\frac{\sqrt{3}}{2})}{a(-\frac{1}{2})+b(\frac{1}{2})} = -\frac{1}{\sqrt{3}}$$

$$\frac{-\sqrt{3}(a+b)}{-a+b} = -\frac{1}{\sqrt{3}} \xrightarrow{\times(-a+b)} 3(a+b) = a+b$$

$3a+3b = -a+b \to 2a+b=0 \to \frac{b}{a} = -2$ ✓ , $b=-2a$

(۲)

②

$x^2+y^2=1 \longrightarrow x=\frac{1}{2} \rightsquigarrow \frac{1}{4}+y^2=1 \longrightarrow y=\pm\frac{\sqrt{3}}{2}$

$P(\frac{1}{2}, +\frac{\sqrt{3}}{2}) \longrightarrow 60 \rightsquigarrow 60+210=270$

$P(\frac{1}{2}, -\frac{\sqrt{3}}{2}) \longrightarrow 300 \rightsquigarrow 300+210=150$

(۲)

حالت A)

$P=(\frac{1}{2}, \frac{\sqrt{3}}{2}) \longrightarrow P'=(0,-1)$ $\quad PP' = \sqrt{(\frac{1}{2})^2+(\frac{\sqrt{3}}{2}+1)^2} \Rightarrow \sqrt{2+\sqrt{3}}$

:

$P(\frac{1}{2}, -\frac{\sqrt{3}}{2}) \longrightarrow P'(-\frac{\sqrt{3}}{2}, \frac{1}{2})$ $\quad PP' = \sqrt{(\frac{1}{2}+\frac{\sqrt{3}}{2})^2+(-\frac{\sqrt{3}}{2}-\frac{1}{2})^2} = \frac{1+\sqrt{3}}{\sqrt{2}} = \sqrt{2+\sqrt{3}}$

محیط مثلث $\to OP+OP'+PP' = 1+1+\sqrt{2+\sqrt{3}} = 2+\sqrt{2+\sqrt{3}}$ ✓

③ در هر ساعت عقربه ساعت شمار $\frac{\pi}{6}$ می‌چرخد

$$\frac{\frac{\pi}{9}}{\frac{\pi}{6}} = \frac{2}{3} \longrightarrow \frac{2}{3}\times 60 = 40$$ ✓

(۲)

④

$|30h - \frac{11}{2}m| \Rightarrow |90 - \frac{11}{2}t| = 20$ ($h=3$)

$90-\frac{11}{2}t=20 \longrightarrow t=12.72$ بار اول

$90-\frac{11}{2}t=-20 \longrightarrow t=20$ ✓ بار دوم

(۲)

⑤

$-\frac{1}{2} \leq \frac{m-3}{5} \leq 1 \longrightarrow +\frac{1}{2} \leq m \leq 8$ ✓ $\xrightarrow{\text{اعداد صحیح}} 1,2,3,4,5,6,7,8 \longrightarrow$ ✓ ۸ تا

(۲)

⑥

$\sin 105 = \cos 15$ $\quad 270-15$ $\quad -\sin 15$

$\cos 255 = \cos(~~180+75~~) = ~~-\cos 75~~$

$\cos 165 = -\cos 15$ $\quad \sin 195 = -\sin 15$

$$\frac{\cos 15 - K ~~\cos 75~~ \sin 15}{-\cos 15 + 2(-\sin 15)} = 3$$

(۱)

$\to$ ~~$\cos 15 - K\cos 75 = -3\cos 15 + 4\sin 15 \to K = 4\cos 15 - 4\sin 15$~~

$\tan 15 = \frac{25}{100} = \frac{1}{4} = \frac{\sin 15}{\cos 15}$ ~~$\cos 15$~~

$$\frac{\cos 15 - K\sin 15}{-\cos 15 - 2\sin 15} \div \cos \longrightarrow \frac{4-K(1)}{-4-2} = 3 \longrightarrow 4-K=-6 \to K=10$$

$$\frac{1-K\tan 15°}{-1-2\tan 15°} = 3 \longrightarrow 1-\frac{K}{4} = -3-\frac{6}{4} \longrightarrow -\frac{K}{4} = -\frac{22}{4}$$

$K=22$

(سارا امینی راد)

۷) $\frac{\cot x + \cot x}{\tan x - 2\tan x} = \frac{2\cot x}{-\tan x} = 2\left(\frac{\frac{\cos x}{\sin x}}{-\frac{\sin x}{\cos x}}\right) = -2\frac{\cos^2 x}{\sin^2 x} = -2\left(\frac{1}{9}\right) = -\frac{2}{8}$

$1 - \cos x = 2 + 2\cos x$

$-1 = 3\cos x \to \cos x = -\frac{1}{3}$

$\sin x = \sqrt{1-\cos^2 x}$

$\sqrt{1-\frac{1}{9}} = +\frac{\sqrt{8}}{3}$

۴) $-\frac{2}{\frac{8}{9}} = -\frac{2}{8} = \left(\frac{1}{4}\right)$ ✓

۸) $\tan = \frac{\tan\pi}{10} = \frac{3\pi}{5}$ $\quad 2A \times \frac{3\pi}{5} = 2B \times \frac{9\pi}{10} \to \frac{2A}{2B} = \frac{\frac{9\pi}{10}}{\frac{3\pi}{5}} = \frac{\frac{9}{10}}{\frac{3}{5}} = \frac{3}{2}$

$2B = 16$

۴) $\text{مساحت} = \left(\frac{2A}{2B}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$ ✓

۹) $AB \Rightarrow h = \tan 60 = \sqrt{3}$ $\quad AB = \sqrt{3}$

$\frac{1}{2} \times AB \times BC = \sqrt{3}$ $\quad AC = \sqrt{AB^2 + BC^2} = \sqrt{3+4} = \sqrt{7}$

$\hookrightarrow BC = 2$

$\sin 2C \to 2\sin C\cos C \to 2 \times \sqrt{\frac{3}{7}} \times \sqrt{\frac{4}{7}} = 2\sqrt{\frac{12}{49}}$ ✓

۴)

$\sin C = \sqrt{\frac{3}{7}}$ $\quad \cos C = \sqrt{1-\sin^2 C} = \sqrt{1-\frac{3}{7}} = \sqrt{\frac{4}{7}}$

۱۰) $B = (0,2)$ / $C = (3,2)$ / $D = (3,0)$ / $A = (0,0)$ $\quad |2\times 2 - 3\times 3|$

$AM = (2,2)$ $\quad AN = (3,2)$

$\sin\theta = \frac{3}{\sqrt{13}\sqrt{13}} = \frac{3}{13}$ ✓

$\sqrt{3^2+2^2}$

۴)