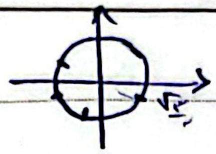
① $\frac{a\sin\frac{4\pi}{3} + b\cos\frac{7\pi}{6}}{a\sin\frac{11\pi}{6} + b\cos\frac{\pi}{3}} = \tan\frac{5\pi}{6}$

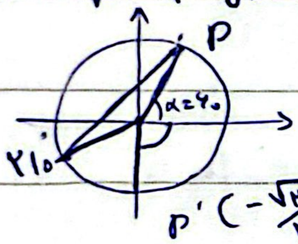


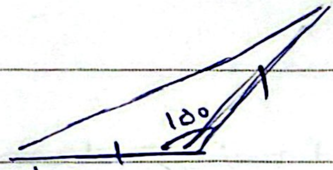
$\rightarrow \frac{a(-\frac{\sqrt{3}}{2}) + b(-\frac{\sqrt{3}}{2})}{a(-\frac{1}{2}) + b(\frac{1}{2})} = -\frac{\sqrt{3}}{3} \rightarrow \frac{\frac{\sqrt{3}}{2}(a+b)}{\frac{1}{2}(b-a)} = -\frac{\sqrt{3}}{3}$

$3a + 3b = b - a \longrightarrow 4a = -2b \longrightarrow \frac{b}{a} = -\frac{4}{2} = -2$

② $P(\frac{1}{2}, y) \xrightarrow{\text{دایره مثلثاتی}} P(\frac{1}{2}, \frac{\sqrt{3}}{2}) \longrightarrow \alpha = 60^\circ$

$(\frac{1}{2}, \frac{\sqrt{3}}{2})$





$P'(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$PP'^2 = ((\frac{1}{2}+\frac{\sqrt{3}}{2})^2 + (\frac{1}{2}+\frac{\sqrt{3}}{2})^2)$

$PP'^2 = \frac{1}{4} + \frac{3}{4} + \frac{\sqrt{3}}{2} + \frac{1}{4} + \frac{3}{4} + \frac{\sqrt{3}}{2}$

$PP'^2 = 1 + \frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{3}}{2} = 2 + \sqrt{3}$

$\longrightarrow 2P = 1 + 1 + \sqrt{2+\sqrt{3}} = \boxed{2 + \sqrt{2+\sqrt{3}}}$

③ ~~[crossed out]~~ $\frac{\frac{\pi}{9}}{\pi} = \frac{x}{180} \quad x = 20^\circ$

~~[crossed out]~~ $\xrightarrow[\text{۲۰ دقیقه جابجا شده}]{\text{دقیقه شمار}}$ $2:30'$

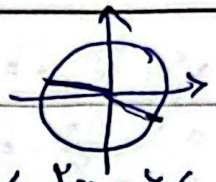
④ $|5.5M - 30h| = 20$

$|5.5M - 90| = 20$

$\rightarrow 5.5M = 110 = \boxed{20\,min}$

$\rightarrow 5.5M = 70 \sim M = 12.7$ ⊗

۷) $-\frac{\pi}{۲} < x < \frac{۵\pi}{۲}$, $\sin x = \frac{m-۳}{۵}$



$-\frac{۱}{۲} < \sin x \leq ۱ \quad ; \quad -\frac{۱}{۲} < \frac{m-۳}{۵} \leq ۱ \longrightarrow -۵ < ۲m - ۶ \leq ۵$

$۱ < ۲m \leq ۱۶ \longrightarrow \frac{۱}{۲} < m \leq ۸$

۶) $\frac{\sin(۱۰۵) + k\cos(۲۵۵)}{\sin(۱۶۵) + ۲\sin(۱۹۵)} = ۳$ $\quad \tan ۱۵^\circ = ۰/۲۶$

$\frac{\cos ۱۵ - k \sin ۱۵}{\sin ۱۵ - ۲\sin ۱۵} = ۳ \qquad \frac{۱ - k\tan ۱۵}{-\tan ۱۵} = ۳$

$۱ - k\tan ۱۵ = -۳\tan ۱۵ \qquad ۱ = (k-۳)\tan ۱۵ \quad \frac{۱}{۲}$

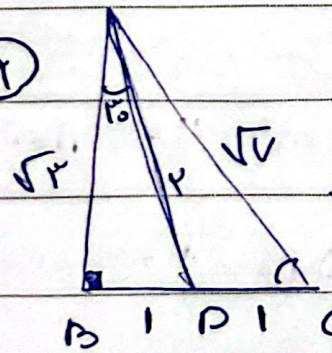
$\longrightarrow k = ۷$

۸) $A = \frac{۱۵\pi}{۱۸۰} = \frac{x}{۳۶۰} \qquad x = \frac{۳۶\pi}{۵} \qquad l = \frac{۳۶\pi}{۵} \times R$

$B = \qquad\qquad l = \frac{۹\pi}{۵} \times R'$

$\longrightarrow \frac{۳۶}{۵} \times R_a = \frac{۹}{۵} \times R'_b \longrightarrow \frac{R_a}{R_b} = \frac{۹}{۳} = ۳$ ✓ $\longrightarrow$ ۹

۹)



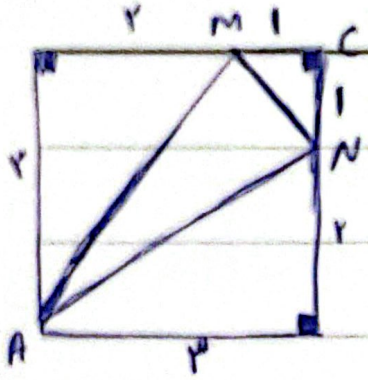
$\sin ۲C = ?$

$\frac{\sqrt{۳} \times BC}{۲} = \sqrt{۳} \longrightarrow BC = ۲$

$\sin ۲C = ۲\sin\hat{C}\cos\hat{C}$

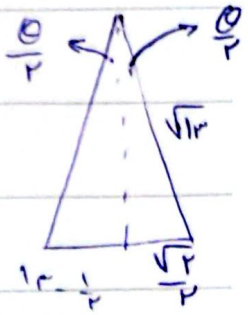
$= ۲ \times \frac{\sqrt{۳}}{\sqrt{۷}} \times \frac{۲}{\sqrt{۷}} = \frac{۴\sqrt{۳}}{۷}$

Arman



$NA = MA = \sqrt{4+9} = \sqrt{13}$

$MN = \sqrt{1+1} = \sqrt{2}$



$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\sin\theta = 2\left(\frac{\sqrt{2}}{2\sqrt{13}}\right) \times \left(\frac{\sqrt{25}}{\sqrt{2}}\right) = \frac{\sqrt{25}}{\sqrt{13}} = \frac{5}{\sqrt{13}}$

$= \frac{5\sqrt{13}}{13}$

Arman