نام: ثمین چنانی   کلاس: دوازدهم دختر A   شماره ۱۲

جواب سوال ①

$\sin \frac{4\pi}{3} = \frac{-\sqrt{3}}{2}$ $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ $\sin \frac{11\pi}{6} = -\frac{1}{2}$ $\cos \frac{\pi}{3} = \frac{1}{2}$

$$\frac{a\left(\frac{-\sqrt{3}}{2}\right) + b\left(\frac{-\sqrt{3}}{2}\right)}{a\left(-\frac{1}{2}\right) + b\left(\frac{1}{2}\right)} = \frac{-\frac{\sqrt{3}}{2}(a+b)}{-\frac{1}{2}(a-b)} = \frac{\sqrt{3}(a+b)}{(a-b)} = \frac{-\sqrt{3}}{3}$$

$$\frac{a+b}{a-b} = -\frac{1}{3} \longrightarrow b-a = 3a+3b \longrightarrow -4a = 2b \longrightarrow a = -\frac{1}{2}b$$

$$\frac{b}{a} = \frac{b}{-\frac{1}{2}b} = -2$$

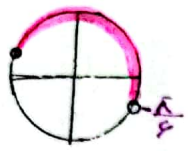
جواب سوال ③ به ازای هر ساعت (۶۰ دقیقه) ۳۰ درجه عقربه ساعت‌شمار می‌کند.

$\frac{\pi}{9} = 20^\circ$

20°	$60\ min$
30°	$?\ min$

$\longrightarrow$ بنابراین ساعت 240° می‌شود.

جواب سوال ⑤

$-\frac{\pi}{6} < x \leq \frac{5\pi}{6} \Rightarrow$ 

$\longrightarrow -\frac{1}{2} < \sin x \leq 1 \longrightarrow -\frac{1}{2} < \frac{m-3}{5} \leq 1$

I) $-\frac{1}{2} < \frac{m-3}{5} \longrightarrow -1 < \frac{2m-6}{5} \longrightarrow -5 < 2m-6 \longrightarrow 2m-6 < 5 \longrightarrow 2m-11 < 0$

$m \in (-\infty, \frac{11}{2})$

II) $\frac{m-3}{5} \leq 1 \longrightarrow m-3 \leq 5 \longrightarrow m-8 \leq 0 \longrightarrow m \in (-\infty, 8]$

$I \cap II \longrightarrow m \in (-\infty, \frac{11}{2})$

جواب سوال ①

$108 \times \frac{\pi}{180} = \frac{3\pi}{5}$

$\frac{3\pi}{5} r_A = \frac{9\pi}{10} r_B \longrightarrow r_A = \left(\frac{9\pi}{10} \times \frac{5}{3\pi}\right) r_B \longrightarrow r_A = \frac{3}{2} r_B$

$\frac{S_A}{S_B} = \frac{\frac{9}{4} r_B^2 \pi}{r_B^2 \pi} = \frac{9}{4}$

جواب سوال ④ زاویه بین عقربه‌های ساعت‌شمار و دقیقه‌شمار:

$\alpha = |5,5m - 30h|$

$20 = |\frac{11}{2} m - 90| \Rightarrow \frac{11}{2} m - 90 = \pm 20$

$\rightarrow m = \frac{140}{11} \simeq 13$

$\rightarrow m = 20$ ✓

بعد از گذشت ۲۰ دقیقه

جواب سوال ۶

$$\frac{\sin(\frac{\pi}{2}+\theta)+k\cos(\frac{3\pi}{2}-\theta)}{\cos(\pi-\theta)+2\sin(\pi+\theta)} = \frac{\cos\theta - k\sin\theta}{-\cos\theta - 2\sin\theta} \xrightarrow{\text{صورت و مخرج} \div \cos}$$

$$\frac{1-k\tan\theta}{-1-2\tan\theta} \xrightarrow{\tan\theta=\frac{1}{4}} \frac{\frac{4-k}{4}}{-\frac{3}{2}} = \frac{4-k}{4}\times\left(\frac{-2}{3}\right) \Rightarrow \frac{-8+2k}{12}=3$$

$$-8+2k=36 \longrightarrow 2k=44 \longrightarrow \boxed{k=22}$$

جواب سوال ۷

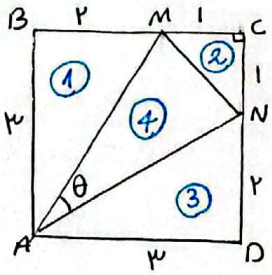
$$\frac{1-\cos x}{1+\cos x}=2 \longrightarrow 1-\cos x = 2+2\cos x$$

$$-3\cos x = 1 \longrightarrow \cos x = -\frac{1}{3} \longrightarrow \sin^2 x+\cos^2 x=1 \longrightarrow \sin^2 x+\frac{1}{9}=1$$

$$\sin x = \pm\frac{2\sqrt{2}}{3} \xrightarrow{\text{ناصیه ۲}} \frac{2\sqrt{2}}{3} \qquad \tan = \frac{2\sqrt{2}}{3}\times\frac{3}{-1}=2\sqrt{2} \qquad \cot=\frac{\sqrt{2}}{4}$$

$$\frac{\tan(\frac{\pi}{2}-x)-\cot(\pi-x)}{\cot(\frac{3\pi}{2}-x)+2\tan(2\pi-x)} = \frac{\cot x+\cot x}{\tan x-2\tan x} = \frac{\frac{\sqrt{2}}{2}}{-2\sqrt{2}} = \frac{\sqrt{2}}{2}\times\frac{1}{-2\sqrt{2}} = \boxed{-\frac{1}{4}}$$

جواب سوال ۱۰



$$S_{\text{مربع}} = 3^2 = 9$$

$$S_{\triangle ①} = \frac{1\times 1}{2} = \frac{1}{2} \qquad S_{\triangle ③,④} = \frac{2\times 3}{2}(2) = 6$$

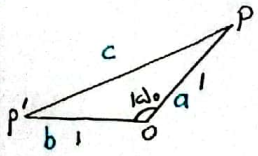
$$S_{\triangle ④} = \frac{18}{2}-\frac{13}{2} = \frac{5}{2}$$

$$AN = AM = \sqrt{2^2+3^2} = \sqrt{13}$$

$$S_{\triangle ④} = \frac{1}{2}(\sqrt{13})(\sqrt{13})\sin\theta = \frac{13}{2}\sin\theta = \frac{5}{2} \longrightarrow \boxed{\sin\theta = \frac{5}{13}}$$

جواب سوال ۲

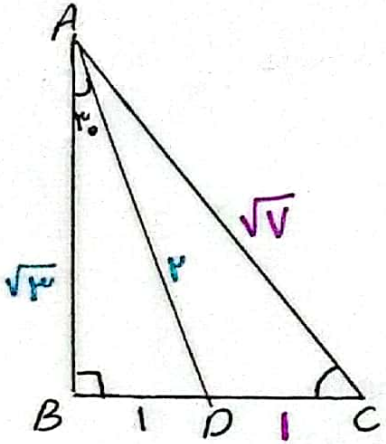
$P(\frac{1}{2}, y) \longrightarrow y=\frac{\sqrt{3}}{2}$ (یک نزم نبود)



$$c=\sqrt{a^2+b^2-2ab\cos 135^\circ} = \sqrt{1+1-2(-\frac{\sqrt{2}}{2})} = \sqrt{2+\sqrt{2}}$$

$$\text{محیط } \triangle OPP' = 1+1+\sqrt{2+\sqrt{2}} = \boxed{2+\sqrt{2+\sqrt{2}}}$$

جواب سوال ⑨ :



$$S_{\text{مثلث } ABC} = \frac{۱}{۲}(\sqrt{۳})(۱)(۱+DC) = \sqrt{۳}$$

$$\boxed{DC = ۱}$$

$$AC = \sqrt{BC^۲ + AB^۲} = \sqrt{۴ + ۳} = \sqrt{۷}$$

$$\sin^۲ \hat{C} = ۲\sin C \cdot \cos C = ۲\left(\frac{\sqrt{۳}}{\sqrt{۷}}\right)\left(\frac{۲}{\sqrt{۷}}\right) = \boxed{\frac{۴\sqrt{۳}}{۷}}$$