20

نرگس روزبه‌دل

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$\hat{A}+\hat{B}+\hat{C}=180^\circ \Rightarrow \hat{A}=60^\circ \qquad a=BC=20 \qquad b=AC=\frac{20\sqrt{6}}{3}$

$\hat{B}+\hat{C}=120^\circ \Rightarrow 180^\circ-120^\circ=60^\circ \qquad \frac{20}{\sin 60^\circ}=\frac{20}{\frac{\sqrt{3}}{2}}=\frac{40}{\sqrt{3}} \Rightarrow \frac{\frac{20\sqrt{6}}{3}}{\sin\hat{B}}=\frac{40}{\sqrt{3}}$

$\Rightarrow \frac{a}{\sin\hat{A}}=\frac{b}{\sin\hat{B}}=\frac{c}{\sin\hat{C}}$

$\Rightarrow \sin\hat{B}=\frac{20\sqrt{6}}{3\sqrt{3}}\times\frac{\sqrt{3}}{40}=\frac{\sqrt{2}}{2} \Rightarrow \hat{B}=45^\circ \Rightarrow \hat{C}=180-(\underbrace{45^\circ+60^\circ}_{105})=75^\circ \qquad 120-45=75^\circ$

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$180-(\underbrace{60+75}_{135})=45^\circ \Rightarrow \frac{a}{\sin\hat{A}_1}=\frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}}=4 \Rightarrow \frac{AC}{\sin 60^\circ}=4 \Rightarrow AC=2\sqrt{3}$

$\frac{AC}{\sin\hat{B}}=\frac{2\sqrt{3}}{1}=\frac{x}{\sin\hat{A}} \Rightarrow a=2\sqrt{3}\times\frac{\sqrt{3}}{2}=3$

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$\frac{c}{\sin\hat{C}}=\frac{b}{\sin\hat{B}} \Rightarrow \frac{\sin\hat{C}}{\sin\hat{B}}=\frac{c}{b} \Rightarrow (b^2-2)\left(\frac{c}{b}\right)=c \Rightarrow b^2-2=b \Rightarrow b^2-b-2=0$

$\Rightarrow (b-2)(b+1)=0 \Rightarrow b=2$

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$b^2=a^2+c^2-2ac\cos\hat{B} \Rightarrow 13^2=21^2+20^2-2\times21\times20\times\cos\hat{B}$

$b \to 13,\ a \to 21,\ c \to 20$

$\Rightarrow 169=\underbrace{441+400}_{841}-840\cos\hat{B} \Rightarrow -672=-840\cos\hat{B} \Rightarrow \cos\hat{B}=\frac{672}{840}=0.8$

$\Rightarrow \sin\hat{B}=0.6 \leftarrow 1-\frac{16}{25}=\frac{9}{25}$

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۵- الف)

$$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$$

$$b^2 \to 16, \quad c^2 \to 4(1+\sqrt{3})^2, \quad \cos\hat{A} \to \frac{1}{2}$$

$$\Rightarrow a^2 = 16 + 4(1+\sqrt{3})^2 - 8(1+\sqrt{3}) = 16 + 16 + \cancel{8\sqrt{3}} - 8 - \cancel{8\sqrt{3}}$$

$$4(1+\sqrt{3})^2 \to 4(4+2\sqrt{3}), \quad -8(1+\sqrt{3}) \to -8-8\sqrt{3}$$

$$\Rightarrow a^2 = 24 \Rightarrow a = \sqrt{24} = 2\sqrt{6}$$

ب)

$$\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}}$$

$$\Rightarrow \frac{\sqrt{24}}{\frac{\sqrt{2}}{2}} = \frac{2\sqrt{6}}{\frac{\sqrt{2}}{2}} = 4\sqrt{2} = \frac{4}{\sin\hat{B}} \Rightarrow \sin\hat{B} = \frac{4}{4\sqrt{2}} \Rightarrow \hat{B} = 45^\circ$$

$$C = 180 - (\underbrace{45^\circ + 60^\circ}_{105}) = 75^\circ$$

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$BC = AC \Rightarrow \hat{\alpha} = \hat{B} \Rightarrow \hat{C} = 180 - (2\alpha)$ $\qquad \frac{c}{\sin \hat{C}} = \frac{BC}{\sin \hat{\alpha}} \Rightarrow \frac{1}{\sin \hat{C}} = \frac{2}{\sin \hat{\alpha}}$

$\Rightarrow 2\sin \hat{C} = \sin\alpha \Rightarrow 2\sin(180 - 2\alpha) = \sin\alpha \Rightarrow 2\sin 2\alpha = 2(2\cancel{\sin\alpha}\cos\alpha) = \cancel{\sin\alpha}$

$\Rightarrow \cos\alpha = \frac{1}{4}$

$\Rightarrow DE = \sqrt{36 + 16 - 48 \times \frac{1}{4}} = \sqrt{52 - 12} = \sqrt{40} = 2\sqrt{10}$

$a^2 = b^2 + c^2 - 2bc \cos \hat{A}$

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۷- الف)

$a^2 = b^2 + c^2 - 2bc\cos\theta$ $\qquad BC = \sqrt{64 + 25 - 80 \times \frac{1}{2}} = \sqrt{49} = 7$

ب)

$S_{BAC} = 8 \times 5 \sin 60 \times \frac{1}{2} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$

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$(b+c)^2 = b^2 + c^2 + 2bc$

$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

$\Rightarrow \frac{\cancel{b^2} + \cancel{c^2} + 2bc - \cancel{b^2} - \cancel{c^2} + 2bc\cos\hat{A}}{bc(\cos\hat{A} + 1)} = \frac{2bc(\cos\hat{A} + 1)}{bc(\cos\hat{A} + 1)} = 2$

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$\cancel{a^3}+b^3-c^3=\cancel{a^3}+ba^2-ca^2 \Rightarrow \cancel{(b-c)}(b^2+c^2+bc)=a^2\cancel{(b-c)} \Rightarrow a^2=b^2+c^2+bc$ ۹-

$a^2=b^2+c^2-2bc\cos\hat{A} \Rightarrow \cancel{b^2}+\cancel{c^2}-2bc\cos\hat{A}=\cancel{b^2}+\cancel{c^2}+bc \Rightarrow -2b\cancel{c}\cos\hat{A}=b\cancel{c}$

$\Rightarrow \cos\hat{A}=-\frac{1}{2} \longrightarrow \boxed{\hat{A}=120^\circ}$

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$S_{ABC}=(3-\cos\theta)(1+3\cos\theta)\sin 30^\circ \times \frac{1}{2}=2$, $\quad \sin 30^\circ \to \frac{1}{2}$

$\Rightarrow 3+9\cos\theta-\cos\theta-3\cos^2\theta=8$ ۱۰-

$\Rightarrow 3\cos^2\theta-8\cos\theta+5=0 \longrightarrow \cos\theta=+1$, $\quad \cos\theta=+\frac{c}{a}=+\frac{5}{3}$

$\Rightarrow a^2=b^2+c^2-2bc\cos\hat{A}$, $\quad \hat{A} \to 30^\circ$

$\Rightarrow a^2=4+16-16\times\frac{\sqrt{3}}{2}$

$\Rightarrow a^2=20-8\sqrt{3}$

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