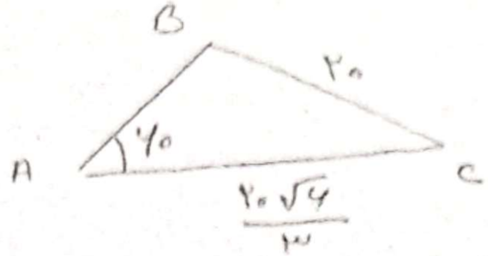
نمره: ۱۹

(1)

$B+C=120$ $\quad A+B+C=180 \longrightarrow A=60^\circ$



$\frac{\frac{20\sqrt{6}}{3}}{\sin B} = \frac{20}{\sin A} = \frac{c}{\sin C}$, $\quad \sin A = \frac{\sqrt{3}}{2}$

$\downarrow$

$\frac{20\sqrt{6}\times\sqrt{3}}{6\times 20} = \frac{\sqrt{18}}{6} = \frac{3\sqrt{2}}{6}$

$\hookrightarrow \frac{\sqrt{2}}{2} \longrightarrow \boxed{B=45}$ ✓

$45+C=120$

$\boxed{C=75}$ ✓

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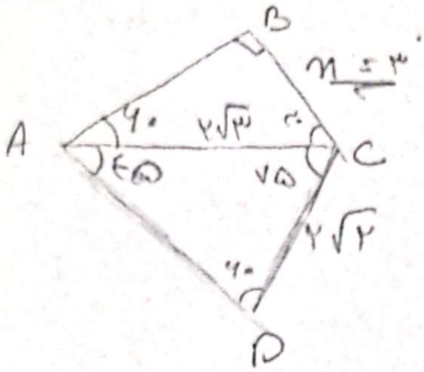
(2)



$n=30$

$60+75=135 \qquad 180-135=45$

$\frac{\sin A}{A} = \frac{\sin D}{D}$

$\longrightarrow \frac{\frac{\sqrt{2}}{2}}{2\sqrt{2}} = \frac{\frac{\sqrt{3}}{2}}{O} \rightarrow \frac{\sqrt{2}}{4\sqrt{2}} = \frac{\frac{\sqrt{3}}{2}}{O}$

۲

$\frac{4\sqrt{2}\times\sqrt{3}}{2\times\sqrt{2}} = 2\sqrt{3}$

$\frac{\sin B}{AC} = \frac{\sin A}{BC}$, $\quad AC = 2\sqrt{3}$, $BC = n$, $\sin A = \frac{\sqrt{3}}{2}$ $\longrightarrow \frac{2\sqrt{3}\times\sqrt{3}}{2} = 3$ جواب ✓

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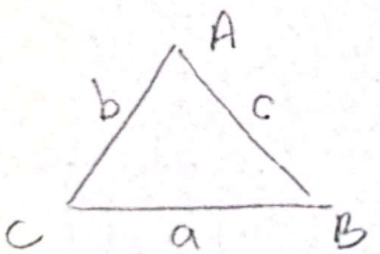
(3)



$(b^2-2)\frac{\sin C^\circ}{\sin B^\circ} = c \qquad b=?$

$\frac{\sin C^\circ}{c} = \frac{\sin B^\circ}{b} \rightarrow \frac{\sin C^\circ}{\sin B^\circ} = \frac{c}{b}$

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$(b^2-2)\times\frac{c}{b} = c \longrightarrow (b^2-2) = c\times\frac{b}{c}$

$b^2-b-2=0 \longrightarrow (b-2)(b+1)=0 \longrightarrow \boxed{b=2}$ ✓

ق ق ۲ $\qquad$ ۱- غ ق ق

۴)

$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

$169 = 49 + 441 - 840\cos\hat{B}$

$\cos\hat{B} = \frac{321}{840} = \frac{4}{5}$ (۲)

$\sin^2 B + \cos^2 B = 1$

$\sin^2\hat{B} + \frac{16}{25} \longrightarrow \sin^2 B = \frac{9}{25} \rightarrow \sin\hat{B} = \frac{3}{5}$

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۵) الف)

$a^2 = 16 + 4(1 + 3 + 2\sqrt{3}) - (16(1+\sqrt{3})\frac{1}{2})$

$a^2 = 16 + 4 + 12 + 8\sqrt{3} - 8 - 8\sqrt{3} = 24 \longrightarrow a = 2\sqrt{6}$

ب)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \longrightarrow \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} = \frac{4}{\sin\hat{B}}$

$\longrightarrow B = 45^\circ$

جمع زوایه های مثلث ۱۸۰ $\longrightarrow a + b + c = 180 \longrightarrow c = 75$ (۲)

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۶)

$a^2 = b^2 + c^2 - 2bc\cos A$

$4 = 4 + 1 - 4\cos\alpha \longrightarrow \cos\alpha = \frac{1}{4}$

$(DE)^2 = 36 + 16 - 48(\frac{1}{4}) \longrightarrow (DE)^2 = 40$

$\longrightarrow DE = 2\sqrt{10}$ (۲)

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۷) الف)

$a^2 = b^2 + c^2 - 2bc\cos A$

$(BC)^2 = 64 + 25 - 40 \rightarrow (BC)^2 = 49 \rightarrow BC = 7$

ب)

مساحت مثلث $= \frac{1}{2} bc\sin\hat{A} \longrightarrow \frac{1}{2} \times 8 \times 5 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$ (۲)

(۸)

$$\frac{(b+c)^2 - a^2}{bc(\cos\hat{A}+1)} = \frac{b^2+c^2+2bc-a^2-b^2+2bc\cos\hat{A}}{bc(\cos A+1)}$$

$$\frac{2bc(\cos\hat{A}+1)}{bc(\cos A+1)} = 2 \checkmark$$

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(۹)

$$\frac{a^3+b^3-c^3}{a+b-c} = a^2 \longrightarrow a^2+b^2-c^2 = a^2+a^2b-a^2c$$

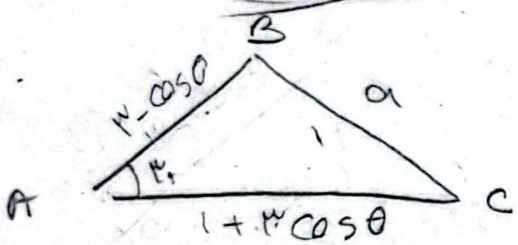
۱٫۵

$$\longrightarrow b^3-c^3 = a^2b-a^2c \longrightarrow (b-c)(b^2+c^2-bc) = a^2(b-c)$$

$$a^2 = b^2+c^2-bc \longrightarrow b^2+c^2-2bc\cos\hat{A} \quad 2\cos A^\circ = -1$$

$$2\cos A = 1 \longrightarrow \cos\hat{A} \ (\tfrac{1}{2}) \longrightarrow -1 \quad \cos A^\circ = -\tfrac{1}{2} \rightarrow A^\circ = 120^\circ$$

(۱۰)



۱٫۵

$$\tfrac{1}{2} \times (1+2\cos\theta)(2-\cos\theta) \times \tfrac{1}{2} = 2.$$

$$\tfrac{1}{2}(2-\cos\theta+4\cos\theta-2\cos^2\theta) = 4$$

$$2-\cos\theta+4\cos\theta-2\cos^2\theta = 8$$

$$3\cos\theta-2\cos^2\theta = 6 \xrightarrow{\cos\theta=t} 2t^2-3t+6 = 0$$

$$t^2-3t+2 = 0 \longrightarrow (t-2)(t-1)$$

$\frac{2}{2}=1$ غ ق $\quad \frac{6}{2}=3$ غ ق

$$a^2 = b^2+c^2-2bc\cos\hat{A} \longrightarrow a^2 = 16+4-16 \longrightarrow a^2 = 4$$

$2\times 4\times\frac{\sqrt{2}}{2}\times 2$ $\quad 4\sqrt{2}$ $\quad a^2 = 20-8\sqrt{2}$