(سارا امینی راد)　　۱۶٫۵

①

$A = 180 - (B+C) = 180 - 120 = 60 \to \hat{A}$

$\frac{BC}{\sin A} = \frac{AC}{\sin B} \to \frac{20}{\sin 60} = \frac{20\sqrt{6}/3}{\sin B} \to \sin B = \frac{1}{2} \to \hat{B} = 30 \quad \hat{C} = 120 - 30 = 90$　۱

$\sin \hat{B} = \frac{\sqrt{2}}{2} \quad \hat{B} = 45^\circ \quad \hat{C} = 75^\circ$

②

$\frac{2\sqrt{2}}{\sin 45} = \frac{AC}{\sin 60} \to \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{AC}{\frac{\sqrt{3}}{2}} \to AC = 2\sqrt{3}$

$\frac{2\sqrt{3}}{\sin 90} = \frac{x}{\sin 60} \to \frac{2\sqrt{3}}{1} = \frac{x}{\frac{\sqrt{3}}{2}} \to x = \frac{\sqrt{3}}{2} \times 2\sqrt{3} = 3$ ✓　۲

③

$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow c = b\frac{\sin C}{\sin B} \quad \sim (b^2-2)\frac{\sin C}{\sin B} = b\frac{\sin C}{\sin B}$

$\to b^2 - 2 = b \to b^2 - b - 2 = 0 \to (b-2)(b+1) = 0$

$b = 2$ ✓ ، -1 غ ق ق　۲

④

$b^2 = a^2 + c^2 - 2ac\cos B$　　جایگذاری $\to (13)^2 = (21)^2 + (20)^2 - 2(21)(20)\cos B$

$a = BC = 21 \quad c = AB = 20 \quad b = AC = 13$　۲　$\hookrightarrow \cos B = 0/8$ ✓

$\sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - 0/64} = 0/6$ ✓

⑤

الف)

$a^2 = b^2 + c^2 - 2bc\cos A \to a^2 = (4)^2 + (2(1+\sqrt{3}))^2 - 2(4)(2(1+\sqrt{3}))\cos 60$

$\to a = \sqrt{24} = 2\sqrt{6}$ ✓　۲

ب)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \to \frac{2\sqrt{6}}{\sin 60} = \frac{4}{\sin B} \to \sin B = \frac{\sqrt{2}}{2} \to \hat{B} = 45$ ✓ $\to \hat{C} = 180 - (60+45) = 75$ ✓

⑥　۵

مثلث $ABC \to 4 + 1 - 2 \times 2 \times 1 \times \cos\alpha = 4 \to \cos\alpha = 1/4$

مثلث $ADE = 36 + 16 - 2 \times 6 \times 4 \times 1/4 = 36 + 16 - 12 = a^2$

$a^2 = 40 \to a = \sqrt{40} = 2\sqrt{10}$

(سارا امینی راد)

(۷) الف)

$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$

$\hookrightarrow BC^2 = (\sqrt{3})^2 + (\sqrt{3})^2 - 2(\sqrt{3})(\sqrt{3})\cos 60^\circ \rightarrow \boxed{BC = \sqrt{3}}$ ✓

(۲)

ب)

$S = \frac{1}{2} ab \sin C \rightarrow \frac{1}{2}(AB)(AC)\sin A \rightarrow \frac{1}{2}(\sqrt{3})(\sqrt{3})\sin 60^\circ = \boxed{\frac{3\sqrt{3}}{4}}$ ✓

(۸)

$a^2 = b^2 + c^2 - 2bc\cos A \xrightarrow{\text{طرفین}} (b+c)^2 - a^2 = (b^2 + 2b + c^2) - (b^2 + c^2 - 2bc\cos A)$

$\hookrightarrow \frac{2bc(1+\cos A)}{b(\cos A + 1)} = 2$ ✓

(۲)

(۹)

$a^2 + b^2 - c^2 = a^2(a+b-c) \rightarrow a^3 + b^2 - c^2 = a^3 + a^2 b - a^2 c$

$\hookrightarrow (b-c)(b^2 + bc + c^2) = a^2(b-c) \rightarrow b^2 + bc + c^2 = a^2$

$\xrightarrow{\text{جای گذاری}} a^2 = b^2 + c^2 - 2bc\cos A \longrightarrow b^2 + c^2 - 2b\cos A = b^2 + bc + c^2$

$\cos A = -\frac{1}{2} \rightarrow$ بنابراین $A = 120^\circ$ ✓

(۲)

(۱۰)

$a^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$

$a^2 = (3 - \cos\theta)^2 + (1 + 2\cos\theta)^2 - 2(3 - \cos\theta)(1 + \cos\theta)\frac{\sqrt{3}}{2}$

$\hookrightarrow a^2 = 10 - 3\sqrt{3} - 8\sqrt{3}\cos\theta + (10 + 6\sqrt{3})\cos^2\theta$

$S = \frac{1}{2}(AB)(AC)\sin A = 2 \rightarrow \frac{1}{2}(3 - \cos\theta)(1 + 2\cos\theta) = 4 \rightarrow \cos\theta = 1$

$\boxed{a^2 = 10 - 8\sqrt{3}}$

(۱٫۵)

$a^2 = 40 - 8\sqrt{3}$

$S_{ABC} = \frac{1}{2} \times \frac{1}{2}(1 + 2\cos\theta)(3 - \cos\theta) = 2$

$8 - 3 - \cos\theta - 2\cos^2\theta + 9\cos\theta + 3\cos^2\theta - 8\cos\theta + 2 = 0$

$\cos^2\theta - 8\cos\theta + 15 = 0$

$\cos\theta = 1 \qquad \cos\theta = 5/3$ ✗